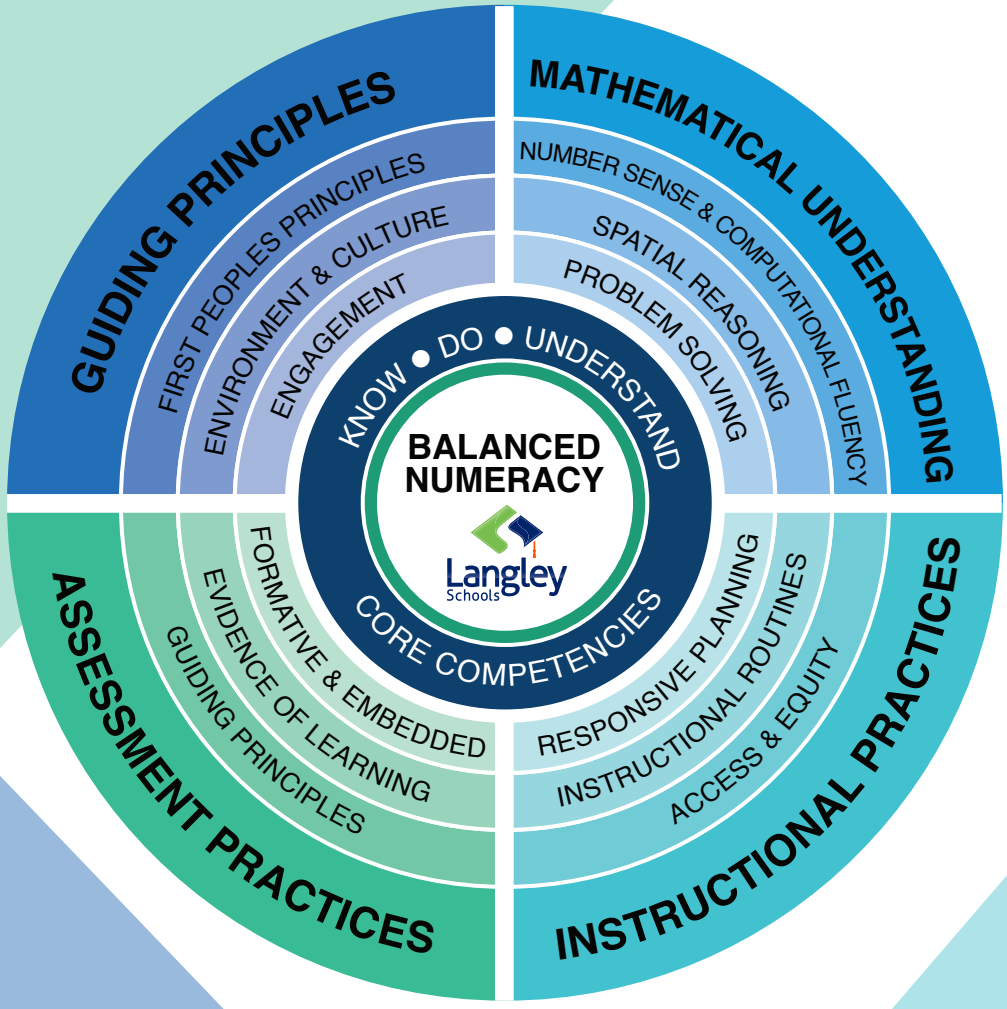
Balanced Numeracy Guide

A RESOURCE FOR GRADE K-8 TEACHERS



CREATED BY TEACHERS FOR TEACHERS

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Instructional Services - District Teachers

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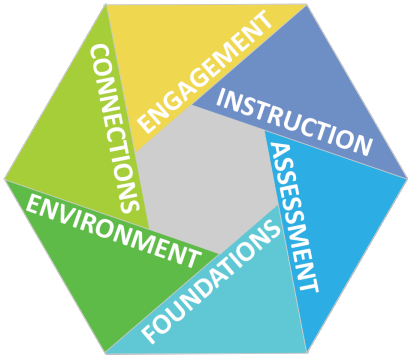
Preface

Message from Instructional Services

Welcome to the Balanced Numeracy Guide, a foundational document for Langley teachers created by Langley teachers. As district teachers, we wanted to develop a supporting document for numeracy instruction similar to our district's Balanced Literacy Guide. Many of our colleagues indicated that they would appreciate a common resource to help develop a balanced numeracy program in their classrooms. Through exploration and conversations, it is our hope that this resource can guide and strengthen your professional learning journey towards ensuring your students experience balanced numeracy.



BC Numeracy Network

Our work was inspired by the Balanced Numeracy website designed by the BC Numeracy Network. Their initial concept, viewpoints, and strategies for effective numeracy instruction were a valuable resource and helped shape the development of this guide.

Message from Dawne Tomlinson, Director of Instruction

We are excited to release the long-awaited K-8 Balanced Numeracy Guide in the Langley School District. Despite several interruptions to our work over the past three years, we have persevered to create this critical document which will anchor our work in numeracy instruction throughout Langley schools. The guide reflects a collaborative process that has involved district teachers, administrators, and classroom teachers who all provided valuable input. The Balanced Numeracy Guide is rooted in evidence-based instructional practice and outlines the foundational pieces required for our learners to become numerate citizens. A huge thank you to Deanna Lightbody, Kim Cline, Shelley Hegedus, and Amanda Slade for their passion and commitment to creating this valuable document.

Acknowledgements

Several people contributed feedback and ideas throughout the developmental process. We would like to thank our colleagues for their time and support.

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We would like to thank Andrea Driedger from Professional Services for her assistance with the design and formatting of this document. Her considerable time and effort are greatly appreciated.

All the resources or suggestions in this guide reflect our own experiences in classrooms and the latest research in mathematical pedagogy. Please see the resources page for references and sources.

Introduction

NUMERACY

What is Numeracy?

Numeracy and literacy are described as the two pillars of the BC curriculum as they are fundamental to all learning. As teachers, we support students to become literate, and it is just as important to help our students become numerate. The BC curriculum defines numeracy as "the willingness and ability to interpret and apply mathematical understanding to solve problems in complex situations, and the perseverance to analyze and communicate these solutions in ways that are relevant to the given context."

At times, the terms numeracy and mathematics are used interchangeably. Numeracy is not the same as mathematics. Mathematics is the discipline, body of knowledge, content, and processes/competencies. Numeracy is applying the mathematics to interpret and understand issues or solve contextual problems. Students need both mathematics and numeracy. Numeracy can be used in all areas of learning. The common goal for the K-12 mathematics curriculum is to develop numerate citizens that use mathematics to make sense of the world around them.

What is Balanced Numeracy?

Balanced numeracy is a framework that incorporates a diverse range of organizational structures, assessments, and instructional practices that are intentional and responsive to students and curriculum. Balanced numeracy provides opportunities for students to uncover, construct, and apply mathematical understandings. Drawing upon the research base for balanced literacy programs, our goal was to create a similar framework to heighten awareness of a balanced numeracy program.

What is the Balanced Numeracy Guide?

The Balanced Numeracy Guide was developed to support teachers with their capacity and confidence in teaching math, with the goal of deeper learning for our students. The Balanced Numeracy Guide is a resource created for teachers by teachers in Instructional Services.

Our Balanced Numeracy Guide Includes:

- Strategies and resources to support the alignment with the BC Math curriculum for Grades K-8.
- A framework that illustrates the necessary components of balanced numeracy in the classroom.
- A "living resource" section with elaborations of numeracy routines, structures, and activities (see OneNote in Teams).
- Support for professional learning that develops increased confidence, understanding, and joy in mathematics education.

BALANCED LITERACY & BALANCED NUMERACY

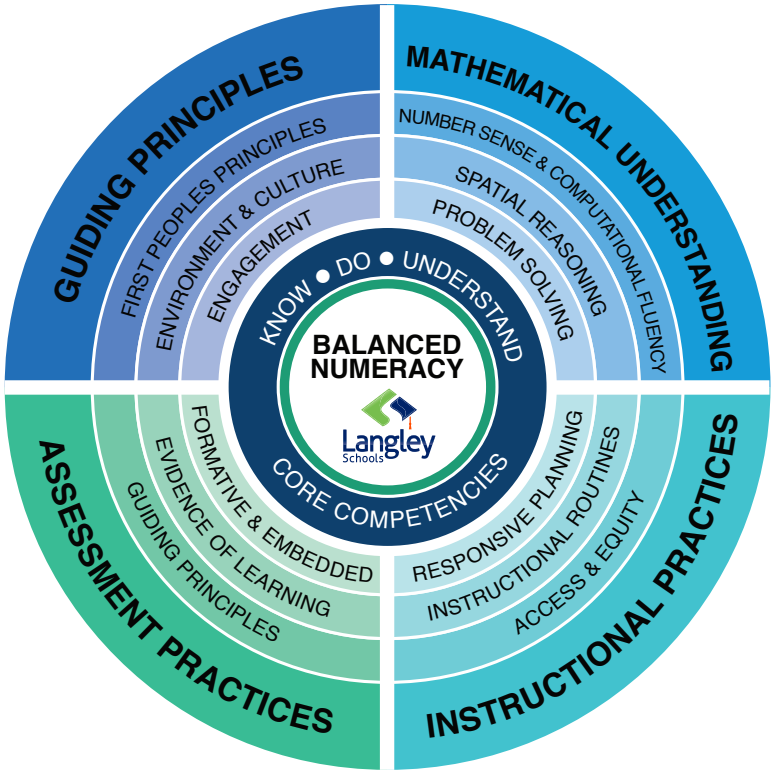
The components of a balanced numeracy approach to teaching mathematics are similar to those within a balanced literacy framework.

Balanced Literacy	Balanced Numeracy
Routines • Transitions (Question of the Day, Morning Message, 'soft start', provocations, etc.)	**Routines** • Daily math investigations • Transitions (Provocations, number sense routines, etc.)
Flexible Groupings • Whole class (Read aloud, mini-lessons, etc.) • Small groups (Mini-lessons, guided reading, literacy centres, literature circles, etc.) • Individual (Conferences, interviews, authentic practice, student choice)	**Flexible Groupings** • Whole class (Modelling, "Number Talks", mini-lessons, explorations) • Small group (Guided math, mini-lessons, math centres, collaborative tasks, problem solving) • Individual (Practice, conferences, interviews)
Explicit and Intentional Planning • Modelling of reading, writing (think aloud) • Shared reading and writing (think-pair-share) • Guided reading and writing • Independent and authentic practice in reading, writing, reflecting	**Explicit and Intentional Planning** • Model (think aloud) • Guided small group practice • Independent exploration and practice
Reading and Writing Workshop (Structure) • Small group • Mini lessons • Conferencing with teacher • Teacher prompting and questioning to encourage further learning	**Math Workshop (Structure)** • Small group • Mini lessons • Conferencing with teacher • Teacher prompting and questioning to encourage further learning
Rich Learning Environment • Classroom library, literacy centres, read aloud, word wall, anchor charts for routines and criteria • Place-based learning, outdoor learning, First Peoples Principles	**Rich Learning Environment** • Math community, student discourse • Choice, play, inquiry, joy, fun • Access to materials (manipulatives, etc.) • Word wall anchor charts, visuals • Rich tasks, problem-solving, inquiry • Visual tools (Visualize, spatial reasoning)
Foundations • Reading and writing strategies • Oral language • Phonological awareness, phonics, sight words, etc.	**Foundations** • Conceptual connections and computational fluency • Concrete, pictorial, abstract—at all grade/age levels (Connected to formal introduction of a concept) • Common language/vocabulary • Compose and decompose quantities, shapes, and fractions • Strategies that demonstrate flexible thinking

HOW TO USE THE BALANCED NUMERACY GUIDE

The guide is designed to support teachers developing a balanced numeracy program in their classroom. We have developed a framework for such a program, represented by the circle graphic below. Each part of the circle represents a component of balanced numeracy: Guiding Principles, Mathematical Understanding, Instructional Practices, and Assessment Practices.

We suggest that you start your professional learning journey by considering which component of balanced numeracy you would like to explore first. Within each component you will find sub-topics or elaborations in which there are considerations or resources included.



Additional Suggestions

- Explore sections or components of the guide at staff meetings.
- Develop a common understanding of the information in the guide.
- Share ideas and strategies.
- Plan or develop common math goals for the school or grade groups.

Important Note

Further elaborations for the components will be added on a regular basis. These elaborations can be found in the Balanced Numeracy OneNote in Microsoft Teams, TCS SS-Grade 3, and TCS Grade 4-8, in the Numeracy channel.

BELIEFS ABOUT TEACHING MATHEMATICS

It is our mission in the Langley School District to inspire student success, confidence, and lifelong learning in mathematics through growth mindsets and innovative teaching. Our district believes that by applying the following principles, we can help ensure mathematical success for all students.

> Math is about sense-making. "This the most fundamental idea that a teacher of mathematics needs to believe and act on. It is through the teacher's actions that every [learner] in his or her own way can come to believe this simple truth and, more importantly, believe that he or she is capable of making sense of mathematics. Helping students come to this belief should be the goal of every teacher."
>
> ***(Van de Walle, Teaching Student-Centred Mathematics, 2006, p. ix)***

Foundational Beliefs

The First Peoples Principles of Learning and other ways of knowing contribute to a more holistic and experiential experience of mathematics and benefit all learners.

All students can learn mathematics with understanding.

Students learn to think like mathematicians by being immersed in the "mathematical habits of mind":

- Persevering and using mathematics to solve problems in everyday life.
- Recognizing there are multiple ways to solve a problem.
- Demonstrating respect for diversity in approaches to solving problems.
- Choosing and using appropriate strategies and tools.
- Pursuing accuracy in problem-solving.

Students learn to be mathematicians by embodying dispositions such as:

- Curiosity and a sense of wonder
- Playfulness
- Flexibility
- Sensemaking
- Resilience

Students learn best when teachers model and nurture a growth mindset, encourage risk-taking in problem-solving, and value mistakes as learning opportunities. Teachers' attitudes can influence students, so it is crucial to represent ourselves as life-long learners of mathematics.

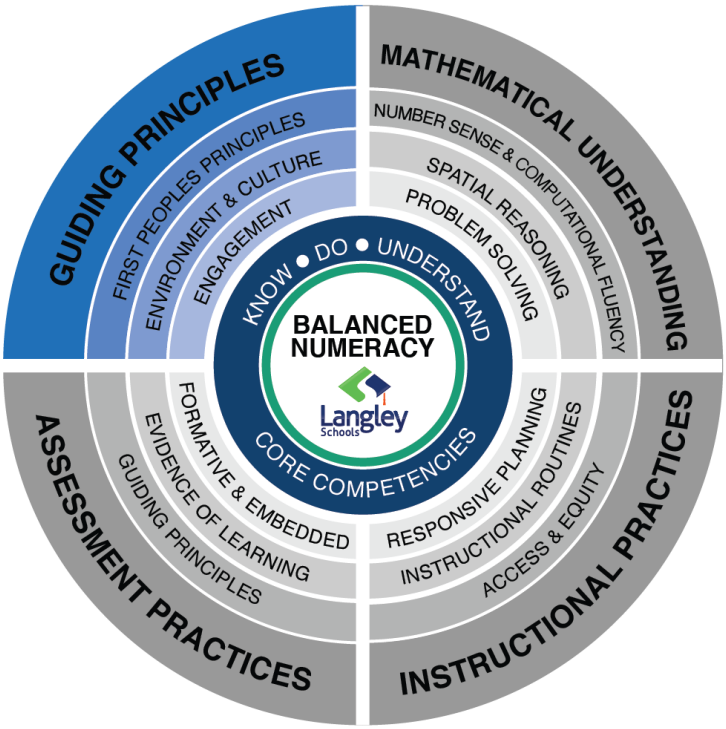
Problem-solving is foundational to the study of mathematics.

Guiding Principles

To explore the foundational beliefs further, this section is divided into three topics: First Peoples Principles of Learning, the classroom environment/culture, and student engagement.

First Peoples Principles of Learning

The First Peoples Principles of Learning emphasize the importance of relationships and interconnectedness in learning. The interconnectedness is not only about connecting learning in a cross-curricular way but also about how learners connect the mathematical skills and curricular competencies to themselves, their families, and communities. There is also a broader sense of relationship that includes one's relationship to land and the natural world. Ultimately, quality math instruction and assessment should embrace learner-centred teaching practice that utilizes connection to land and place, the power of story, and experiential learning opportunities.



Environment & Culture

Classroom design combined with active participation strategies will enhance student learning, increase achievement, and factor in the development of the well-educated citizen (BC Mathematics Curriculum). It is important that students are shown that numeracy is valued in the classroom by creating an environment and culture of numeracy. Teachers whose classrooms make math visible, integrate mathematics throughout the day, and encourage mathematical thinking and communicating can help students embody a positive attitude and appreciation towards mathematics.

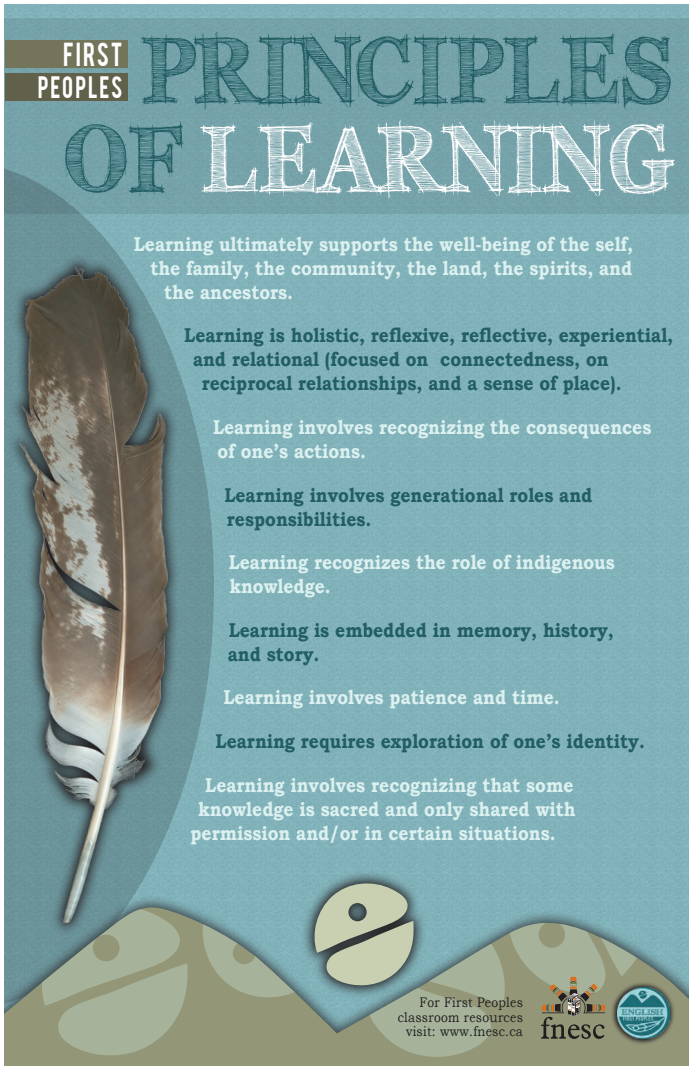
Engagement

Students are engaged in mathematics when provided with developmentally appropriate tasks, encouraged to make meaning of numeracy in the world, and supported in becoming confident mathematicians. It is important to foster joy in mathematics by providing engaging opportunities that provoke and inspire mathematical thinking through play, inquiry, and exploration. Building a community of thinkers in the classroom can help to develop lifelong learners in mathematics.

FIRST PEOPLES PRINCIPLES OF LEARNING

These First Peoples Principles of Learning are thoroughly compatible with approaches to teaching and learning that emphasize:

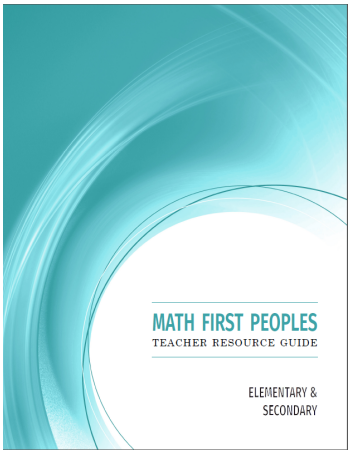
- Building on what students are already familiar with (both abstract knowledge and concrete knowledge).
- Exploring and building on students' interests (e.g., asking learners about what is important to them to identify what context will prove meaningful to them as a basis for learning mathematics).
- Presenting mathematics problems of various sorts in varied ways (e.g., visual, oral, role-play, and experiential problems as well as word and symbol problems).
- Stimulating students' innate curiosity and desire to explore.
- Communicating a positive and enthusiastic attitude toward mathematics (e.g., being willing to take risks and make mistakes and encouraging students to do the same).
- Promoting and rewarding perseverance (e.g., giving necessary time for difficult problems and revisiting them on multiple occasions).
- Encouraging students to reflect on and be explicit about their own thinking processes and the transformations in their own understanding.



Source: FNESC, Math First Peoples Teacher Resource Guide, 2020.

One of the most effective ways to begin teaching Mathematics First Peoples is to establish meaningful connections for students between mathematics skills and "content" and First Peoples themes and topics. To be meaningful, connections must not only be identified at the outset of a teaching unit but must be systematically revisited at appropriate intervals. The tokenism of periodically introducing one-off, trivial examples or contrived problem situations that promote simplistic, stereotypical aspects of First Peoples traditions will be obvious to most students and will likely fail to achieve any meaningful result.

Think about your current mathematical practices. In what ways do these principles come to life? How can you adjust your practice to better embrace them? In what ways do your students recognize these principles?



Visit fnesc.ca/math-first-peoples/ to obtain a copy of the Math First Peoples Teacher Resource Guide.

ENVIRONMENT & CULTURE

Math Anxiety: A Special Challenge in Teaching Mathematics

One of the main challenges in teaching math is the anxiety it can evoke in students and adults. According to professor Daniel Ansari, Canada Research Chair in Developmental Cognitive Neuroscience, math anxiety is pervasive among students, teachers, and parents. Math anxiety can begin as early as Kindergarten, and nearly half of elementary and middle school students experience it. If teachers and parents have experienced anxiety in math, this can influence students' feelings and attitudes towards math and lead to poor math performance. If students are anxious about math, it doesn't mean they can't learn math. There are strategies that teachers can put in place to help students deal with their anxiety and to promote a healthy math identity.

Diminishing Anxiety in Students

There are many ways teachers can decrease the level of anxiety in their students and help them to develop a healthy math identity. The following is a summary of the strategies recommended by Marian Small in her book, Making Math Meaningful, and by Gina Picha in her article, "Recognizing and Alleviating Math Anxiety" (Edutopia, 2017):

- Focus on math as making sense and not as a set of rules that need to be memorized.
- Start from where the students are at or their place of comfort in their ability.
- Use healthy and accurate messages such as "All students can learn math" rather than "Some of us just aren't good at math." Parents and teachers need to be aware of their influence and any negative talk or attitudes they might pass on to students.
- Ask questions that have a variety of solutions or alternative approaches and allow for "think time".
- Assign fewer timed drills or tests.
- Help students develop a "growth mindset" where errors are seen as valuable learning opportunities.

Signs of Math Anxiety

Avoidance
Students with high levels of math anxiety tend to avoid mathematics by showing off task behaviours.

Lack of Response
When students have math anxiety, any math-related question can make them feel extremely stressed. They might not be able to access their working memory, which would make it difficult for them to think clearly. Or some students might be anxious even if they know the answer—it's the fear that is standing in the way, not the math.

Tears or Anger
Sometimes, students with anxiety are very hard on themselves and might feel that being good at math means getting correct answers quickly. This misconception can be harmful for students' math identity and attitudes towards learning new skills.

Negative Self-talk
Students suffering from math anxiety have negative thoughts about math and their own abilities. "I hate math. I'm not good at math. I'll never be able to do this," are typical comments heard from students who are anxious towards math.

Low Achievement
"Math anxiety is not just an emotional problem. It also has cognitive consequences," reports Dr. Ansari. When students experience anxiety, it occupies the working memory, which means more of the brain is dealing with the anxiety and less of it is trying to solve the math problem.

ENVIRONMENT & CULTURE

Place-Based Learning

BC's redesigned curriculum provides teachers with flexibility in creating learning environments that are relevant, engaging, and novel. Flexible learning environments give consideration to local contexts and place-based learning. Math can be seen everywhere. Connecting to our place helps build context to concepts. It embeds relevance and connects students to their worlds.

Learning Outdoors

Experiencing mathematics outdoors can develop mathematical thinking and foster the core competencies. Place-based learning:

- Offers opportunities for mathematical discussion and collaboration.
- Provides problem-solving experiences that are connected to place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures.
- Supports the First Peoples Principle of Learning that learning is holistic, reflexive, reflective, experiential, and relational (focused on connectedness, on reciprocal relationships, and sense of place).
- Lends itself to an inquiry approach to learning and can be used to integrate the various subject areas.
- Helps students connect mathematical concepts to each other and to other areas and personal interests (math to self, math to world, and math to math).

Math can be seen everywhere. Connecting to our place helps build context to concepts. It embeds relevance and connects students to their worlds.

Considerations

- Dress well for the weather and complete a site assessment for safety.
- Set boundaries and behaviour expectations.
- Collect tools to use outdoors: magnifying glasses, clipboards, camera device, measuring tapes, tarps, etc.
- Grouping, patterning, and graphing.

Think of a place or a space around your school. What opportunities are available for students to think about or engage in mathematics?

Examples

- Create counting collections with rocks, sticks, pinecones, or leaves.
- Use natural materials as manipulatives. For example, leaves, rocks, sticks, and shells work well for counting, grouping, patterning, and graphing.
- Measure rain, wind, and temperature.
- Go on focused walks to explore and investigate: Where do we notice patterns, shapes, symmetry, arrays, angles in the world around us? e.g. The points of the maple leaf can be used to exemplify multiplication (2 leaves X 11 points = 22).
- Collect data for monitoring change over time, bird counts, plant growth, etc.

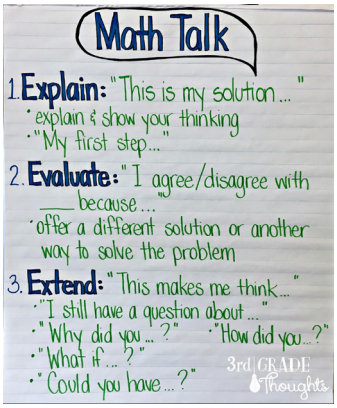
ENVIRONMENT & CULTURE

The Learning Environment

It is important to create a classroom environment that facilitates the learning of numeracy. The design of the learning space, the materials used, and the organization of the classroom can impact the learning environment. By making numeracy "visible" in the classroom, it shows that numeracy is explored and embraced. When students are provided with numeracy experiences beyond the time dedicated for math instruction, it demonstrates the ways numeracy is living throughout the school day and helps students build connections between concepts and subjects.

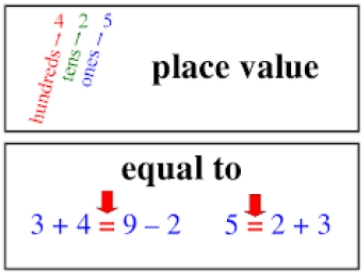
Design Flexible Workspaces

- Classroom set up that facilitates discussion
- Whiteboard space (vertical non-permanent surfaces) for group work and sharing ideas
- Small work areas
- Moveable tables or desks place-based



Use Visual Tools

- Co-created anchor charts
- Ten frame cards
- Posters
- Number lines
- Math word wall



Feature Children's Thinking/Work on Walls

Display Children's Literature Featuring Mathematical Concepts

- Fiction and non-fiction books
- Pictures that inspire mathematical thinking

Make Technology Available to Support Mathematical Explorations

- Rulers, measuring tape, measurement containers
- Calculators and abacus
- Apps that encourage students to represent their thinking and to collaborate

Provide a Variety of Manipulatives that are Accessible

Offer Materials to Help Students Record Their Thinking

- Small whiteboards
- Clipboards and pencils
- Notepads

ENVIRONMENT & CULTURE

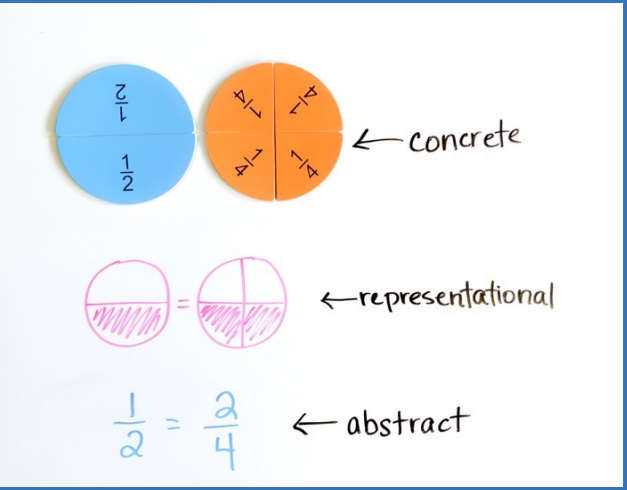
Manipulatives

For decades, educational researchers have recommended the use of manipulatives in teaching mathematical concepts at all grade levels. When students have opportunities to build and represent using manipulatives, they can connect the math ideas and symbols to the physical objects.

This, in turn, helps students gain a better sense of abstract mathematical ideas. The BC Mathematics curriculum suggests that K-12 students have opportunities to represent and communicate their mathematical thinking in concrete and representational forms to help bridge the gap between the abstract or symbolic form.

What is Concrete Representational Abstract?

Teachers are encouraged to use the concrete representational abstract (CRA) instructional approach when introducing new math concepts. The amount of time students need to progress from concrete to abstract understanding varies by concept and by student. There isn't always a linear progression for students. At times, students may be able to move from the concrete representation right to the abstract or symbolic form all in one lesson. The most important thing is that students don't see these phases as different ways to approach a problem, they see them as connected.



Concrete Stage	Representational Stage	Abstract Stage
A mathematical concept is introduced with manipulatives; students explore the concept using the manipulatives in a purposeful activity.	A mathematical concept is represented using pictures of some sort to stand for the concrete objects (manipulatives) of the previous stage; students demonstrate how they can both visualize and communicate the concept at a pictoral level.	Mathematical symbols (numerals, operation signs, etc.) are used to express the concept in symbolic language; students demonstrate the understanding of mathematical concepts using the language of mathematics.

Mathematician Seymour Papert believes manipulatives are "objects to think with."

ENVIRONMENT & CULTURE

Manipulatives: Tips for Teachers

- **Introduce manipulatives**. Talk with kids about why we use manipulatives: They are tools to help us problem solve and learn new math ideas.
- **Allow time for exploration**. Encourage students to explore the materials and make their own discoveries.
- **Establish and review expectations**. Set up and communicate the expectations for distributing, using, and collecting manipulatives in the classroom.
- **Use a variety of manipulatives**. When students can use different manipulatives to represent the same concept, their ability to understand subsequent math concepts is enhanced.
- **Have students work in groups with manipulatives**. This encourages communication and expands students' thinking about how a manipulative can help them solve a problem or serve as a model to represent their ideas.
- **Let parents know**. Inform parents why their children are using manipulatives at times. Try to include manipulative math activities for home.

Calculator Use

NCTM Position

Calculators in the elementary grades serve as aids in advancing student understanding without replacing the need for other calculation methods. Calculator use can promote the higher-order thinking and reasoning needed for problem-solving in our information and technology-based society. Their use can also assist teachers and students in increasing student understanding of and fluency with arithmetic operations, algorithms, and numerical relationships, and enhancing student motivation. Strategic and selective calculator use can aid students in recognizing and extending numeric, algebraic, and geometric patterns and relationships. Teachers should consider how calculator use will support and advance student learning before implementing them in their classrooms.

Note:

- Calculator use does not replace the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.
- Calculators should be implemented strategically in ways that support the development of problemsolving skills.
- Calculator use, when combined with with a mix of instructional styles, does not make students lose their paper and pencil skills.
- Calculator use in the classroom improves the paper and pencil skills of students regardless of their ability levels.
- Those who use calculators in class have better attitudes toward mathematics than children who do not use them.

When and for what purposes should calculators be used in the elementary mathematics classroom?

ENVIRONMENT & CULTURE

Classroom Materials

Every classroom needs to have a core set of manipulatives and materials accessible to students to help support their development in numeracy. The following is a suggested list of materials and manipulatives that can be used across all areas of the K-8 mathematics curriculum.

Manipulatives

- ☐ Attribute Blocks
- ☐ Double Sided Math Counters
- ☐ Snap Cubes
- ☐ Cuisenaire Rods
- ☐ Base 10 Blocks
- ☐ Geometric Shapes (pattern blocks)
- ☐ Fraction Circles
- ☐ Fraction Strips (long strips of paper)
- ☐ Collections (corks, rocks, buttons)
- ☐ Square Coloured Tiles
- ☐ Money Sets
- ☐ Tangram Sets
- ☐ Mirrors
- ☐ Measuring Capacity Set
- ☐ Wooden 3D Models
- ☐ Pentominoes
- ☐ Prisms
- ☐ Weights, stacking mass sets
- ☐ Dominoes

Visual Tools

- ☐ Picture Books
- ☐ Hundreds Charts
- ☐ Multiplication Chart
- ☐ Number Lines (open and numbered)
- ☐ Rulers (metre sticks, measuring tapes)
- ☐ Five Frames and Ten Frames
- ☐ Venn Diagrams
- ☐ Anchor Charts
- ☐ Graph Paper
- ☐ Dotted Paper
- ☐ Place Value Charts
- ☐ T-Charts

Materials for Games

- ☐ Dice (regular, ten-sided, place value, fraction, other)
- ☐ Playing Cards
- ☐ Mini-Whiteboards and markers
- ☐ Plastic Game Sheet Covers
- ☐ Translucent Game Chips
- ☐ Notepads
- ☐ Clipboards

Containers and Supplies

- ☐ Plastic game containers
- ☐ Egg Cartons
- ☐ Ice Cube Trays

ENVIRONMENT & CULTURE

Student-Centred Learning

If you picture a traditional math classroom, think about… who is doing the talking? Who is doing the math? Who is doing the thinking? Chances are the teacher presents the lesson, the students work through problems or questions, and then they might take a quiz to demonstrate understanding. The students may have been doing the math but were probably told how to think and what to do most of the time. Students can become passive learners of mathematics and struggle to connect classroom learning to their own life experiences if they aren't given opportunities to take charge of their own learning and make meaningful connections to the world around them.

Students have different needs, interests, and abilities, so designing mathematical experiences with the success of all learners in mind is essential. Today, teachers understand the need to shift towards creating a student-centred approach for instruction to engage all learners. "Teachers need strategies to encourage student ownership, develop confident math thinkers, and connect math to students' interests and identities" (Nguyen, Edutopia, 2021).

The following is a list of some recommended approaches that can foster a student centred environment:

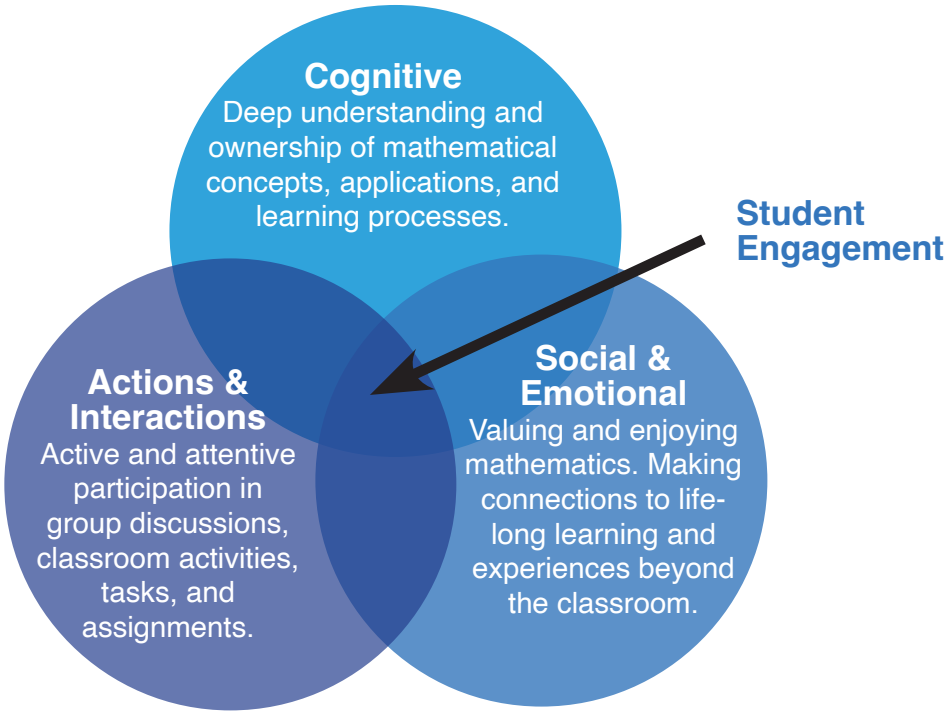
- Know your students.
  - Pre-assessments, webbing, know-wonder-learn, interest surveys, observing and conferencing
- Create a community of learners.
  - A learning environment that builds trust and strong relationships between and among the teacher and students.
  - Encourage students to ask questions to develop critical thinking.
  - Provide opportunities for classroom discourse.
- Build understanding through flexible thinking, reasoning, and problem-solving.
- Connect the math to students' experience when you can.
  - Students' engagement and mathematical understanding deepens when they connect to what is important to them. Find out their interests and where the math lives in it.
- Give students immediate, targeted feedback specific to their learning needs.
- Involve students in setting personal goals for learning.
- Offer students multiple ways so show what they know. Students may choose to use words, pictures, manipulate materials, or even create to express mathematical understanding.
- Encourage productive struggle.
- Allow students to struggle productively as they attempt to solve complex problems.

> "Today, far too many students see mathematics as a subject that must be endured, not a source of inspiration or a way of thinking that can enrich their understanding of the world."
>
> ***American Institutes for Research, 2014***

ENGAGEMENT

Student engagement refers to a students' level of involvement with their learning and is a proven factor in mathematics achievement. It is multidimensional based on a students' active participation, ability to understand the math, and willingness to see the math beyond the classroom. Engagement happens when students are invested in the task at hand, value their learning, and most importantly, have fun with the math.

"Engagement occurs when students are thinking hard, working hard, and feeling good about learning mathematics."
Teller, 2016



Considerations for Developing Student Engagement

- Create emotionally safe classrooms.
- Assess authentically with ongoing feedback.
- Provide engaging opportunities that provoke and inspire mathematical thinking through play, inquiry, and exploration.
- Design interdisciplinary tasks and investigations (STEAM, Math Projects).
- Provide access for all learners through starting points in tasks that are low floor, high ceiling.
- Create flexible, visibly random groupings for group problem-solving.
- Encourage individuality through flexible, open tasks that allow for different approaches and abilities.
- Play math games to practice math facts.
- Provide different tools for students to represent their thinking: vertical spaces, individual white boards, technology, graphic organizers, math journals.
- Foster relationships through structured cooperative or collaborative learning approaches.
- Engage parents and make connections between the classroom and home.

ENGAGEMENT

The Use of Literature in Mathematics

Mathematics is all around us, including in the stories we read and share. Using stories to launch or explore math concepts is uniquely engaging for students.

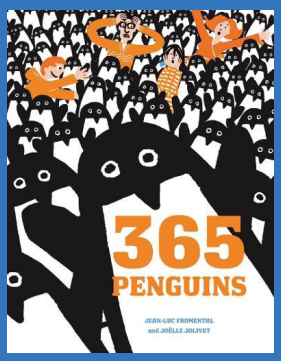
Students can be highly motivated and engaged in learning when using literature—no matter what their grade! Picture books and stories are a natural way to link subjects with mathematics. Stories work as a vehicle to foster curiosity and problem-solving in a new context. Although books can be used to introduce mathematical ideas, there is also value in exploring known math concepts in a meaningful and connected story context. Stories can provide opportunities for students to make mathematics-to-self connections which may increase engagement and learning of mathematics.

Looking through the lens of math can help you see stories and picture books in new ways. Often books that are your teaching favourites in one subject area can become mathematical favourites, too. It is important to remember pictures can also contribute to mathematical understanding and problem-solving. For stories relying on fewer pictures, the literacy strategy of visualizing also contributes to the building of math skills.

Ask Yourself

- Where can I find math in this story?
- How can this story inspire mathematical thinking and problem-solving?
- Do the pictures contribute to mathematical understanding?
- Are there real-life moments in the story that offer opportunities for mathematical inquiry or exploration? i.e. financial literacy and careers in mathematics.
- In what ways can I build literacy and numeracy at the same time?
- How can problem-solving add to the understanding of story?
- What ways can I have my students mathematically respond to this story?
- What are some picture books and stories to teach at my grade level?

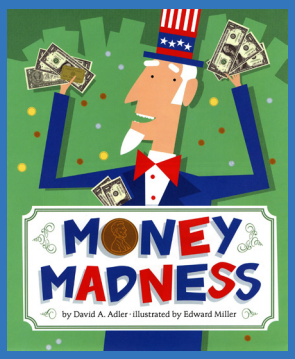
Books That Teach Math



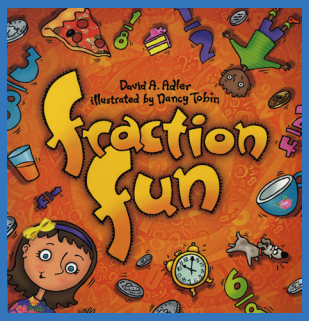
365 Penguins
Jean-Luc Fromental & Joëlle Jolivet

Lion's Share
Matthew McElligott

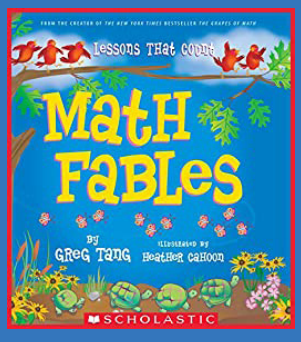
Bean Thirteen
Matthew McElligott



Money Madness
David A. Adler & Edward Miller



Fraction Fun
David A. Adler & Nancy Tobin



Math Fables
Greg Tang & Heather Cahoon

ENGAGEMENT

Play in Mathematics

Play can take many forms and be used to increase student engagement, and develop understanding and application of concepts, as well as to increase automaticity.

1. Inviting students to engage with mathematics through play is essential to their mathematical understanding.
2. Children's engagement and mathematical understanding deepens when they use and manipulate materials.
3. As students routinely use manipulatives mathematical understanding deepens and play becomes more sophisticated.
4. When played repeatedly, games support students' development of computational fluency.

Continuum of Pedagogical Strategies for Play-Based Learning

Child Directed ——— Educator Guided ——— Educator Directed

Free Play	Inquiry	Collaborative Play	Playful Learning	Learning Games
Observe and reflect. Faciliate time, space, and things.	Extend child-initiated ideas and explorations through questions, provocations, investigations.	Use opportunities to enter children's play and incorperate targeted skills into their play.	Focus on targeted skills in learning experiences that can incorporate children's play narrative.	Use opportunities to enter children's play and incorperate targeted skills into their play.

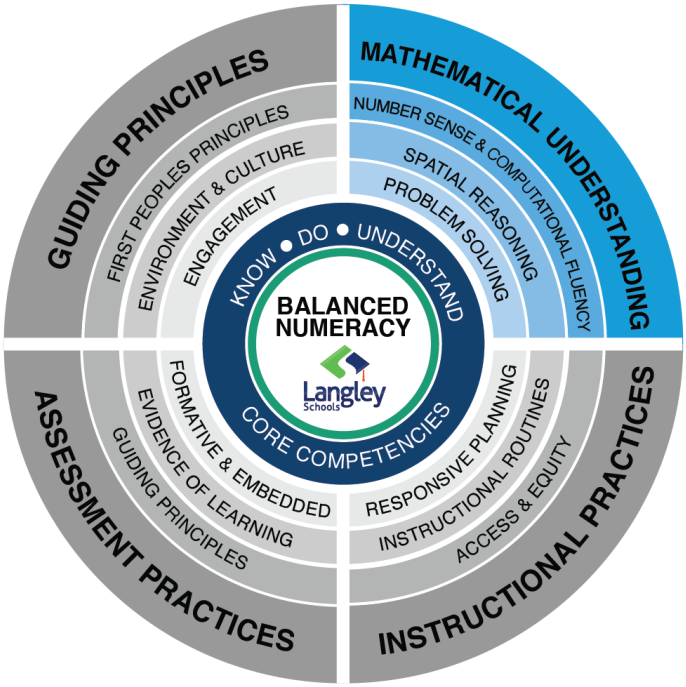
Adapted from Pyle and Danniels, 2017.

Mathematical Understanding

Mathematical understanding refers to the "ability to think and act flexibly with a topic or concept" (Van de Walle, 2014). It is not the ability to memorize and then recite, but rather the ability to explain why and apply this knowledge to other scenarios and tasks.

Number Sense & Computational Fluency

Number sense can be described as "good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (Howden, 1989).



Number sense is the foundational building block of mathematics. It cannot be directly taught; instead, it is developed through various experiences that help students understand number relationships and operations. Students with good number sense can make connections, readily spot patterns, estimate reasonably, and use multiple strategies to solve complex problems.

Computational fluency is defined as having efficient, flexible, and accurate methods for computing (NCTM, 2000). Fluency is developed over time and begins with number sense. Students can develop and improve their fluency through experience and discussion. Students are exposed to different strategies to develop flexibility and efficiency.

Spatial Reasoning

Spatial reasoning is the ability to think about and manipulate objects in three dimensions. Spatial reasoning skills are an essential part of art, physical education, math, and science. We use spatial thinking to understand the location (position) and dimensions (such as length and size) of objects, and how different objects are related to each other. Students' ability for spatial thinking can influence how well they do in mathematics.

Problem-Solving

Problem-solving in mathematics is about working on tasks that stretch and challenge students intellectually for the purpose of strengthening their understanding and development. Problem-solving is not a separate "unit" but rather a part of every mathematics lesson (NCTM). The importance of problem-solving in learning mathematics comes from the belief that mathematics is primarily about reasoning, not memorization. Problem-solving allows students to develop understanding and explain the processes used to arrive at solutions, rather than remembering and applying a set of procedures.

NUMBER SENSE & COMPUTATIONAL FLUENCY

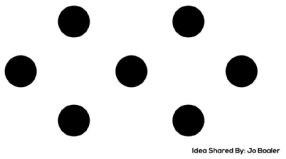
Students' number sense often declines when they are introduced to algorithms—they often become laser-focused on those and stop making sense. Most algorithms are introduced way too early, and they stop sense-making (Boaler, 2018). The focus in the early years on number and computational fluency is imperative in order to build a strong number sense foundation for later mathematical understanding. Students of all grades need hands-on practice multiple times in order to be able to develop number sense.

The Big Ideas in the BC curriculum can be viewed as the goal for mathematical understanding in those areas. Students follow natural developmental progressions (or pathways), and these are the pathways to get to the goals (Big Ideas). Simply put, number sense is the ability to conceptually understand numbers, use numbers flexibly, and apply that knowledge to real-world problems.

The National Council of Teachers of Mathematics identified five components of number sense:

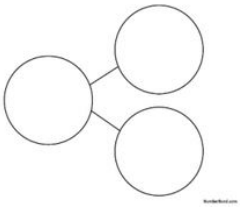
Number Meaning

Children can count out loud to 100 and still not understand the meaning of numbers; they have just memorized a series of words in order. Children need to understand that numbers are symbols that represent quantity. Practice counting, sorting/grouping, and skip counting all serve to help children understand number meaning.



Number Relationships

For students to be able to use numbers flexibly, they need to understand the relationship between numbers. Knowing that 3 and 2 are a part of 5 or to recognize that 7+6 is the double of 6+6 with 1 more shows an understanding of number relationships. Working with decomposing numbers can set students up with mental math skills later on.



Number Magnitude

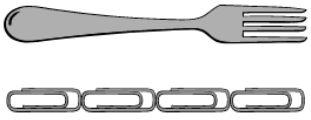
Magnitude refers to the relation of a number or quantity to other objects or items. It is important to teach this to students so that they understand that a number's size depends on what you are comparing it to or where it might fit on a number line along with another number. Children can tell you right away whose candy pile is larger, but can early learners tell where the number 6 fits on a line between 0 and 20?

Number Operations

Once students have a conceptual understanding of number sense through manipulative use and lots of practice counting, sorting, subitizing and decomposing, they will be able to learn how to combine and take away as well as how to group sets for more efficient calculations. These skills are needed to develop computational fluency.

Number Referents

Referents are objects, units of measurement or numbers that can be used to help estimate or to support computational fluency. For example, 1 metre is the distance of a doorknob to the floor. This is helpful when a measuring tool is not available in the real world outside of school. Or, a student might use the benchmarks of 25, 50 and 100 to help with estimation.



NUMBER SENSE & COMPUTATIONAL FLUENCY

Grade	Number	Computational Fluency
K	• Number of concepts to 10 • Ways to make 5 • Decomposition of numbers to 10	• Changed in quantity to 10, using concrete materials • Equality as a balance and inequality as an imbalance
1	• Number concepts to 20 • Ways to make 10	• Addition and subtraction to 20 (understanding of operation and process) • Change in quantity to 20, concretely and verbally • Meaning of equality and inequality
2	• Number concepts to 100 • Benchmarks of 25, 50, and 100, and personal referents	• Addition and subtraction facts to 20 (introduction of computational strategies) • Addition and subtraction to 100 • Change in quantity, using pictorial and symbolic representation • Symbolic representation of equality and inequality
3	• Number concepts to 1000 • Fraction concepts	• Addition and subtraction to 1000 • Addition and subtraction facts to 20 (emerging computational fluency) • Multiplication and division concepts • One-step addition and subtraction equations with an unknown number
4	• Number concepts to 10,000 • Decimals to hundredths • Ordering and comparing fractions	• Addition and subtraction to 10,000 • Multiplication and division of two- or three-digit numbers by one-digit numbers • Addition and subtraction of decimals to hundredths • Addition and subtraction facts to 20 (developing computational fluency) • Multiplication and division facts to 100 (introductory computational strategies) • Algebraic relationships among quantities • One-step equations with an unknown number using all operations
5	• Number concepts to 1,000,000 • Decimals to thousandths • Equivalent fractions • Whole-number, fraction, and decimal benchmarks	• Addition and subtraction of whole numbers and decimals • Multiplication and division of three digits, including division with remainders • Addition and subtraction facts to 20 (extending computational fluency) • Multiplication and division facts to 200 (emerging computational fluency) • One-step equations with variables
6	• Small to large numbers (thousandths and billions) • Factors and multiples • Improper fractions and mixed numbers • Introduction to ratios • Whole-number percents and percentage discounts	• Multiplication and division facts to 100 • Order of operations with whole numbers • Multiplication and division of decimals • One-step equations with whole-number coefficients and solutions
7	• Relationships between decimals, fractions, ratios, and percents	• Multiplication and division facts to 100 (extending computational fluency) • Operations with integers • Operations with decimals • Two-step equations with whole-number coefficents, constants, and solutions
8	• Perfect squares and cubes • Square and cube roots • Percents less than 1 and greater than 100 • Numerical proportional reasoning (rates, ration, proporions, and percent)	• Operations with fractions (addition, subtraction, multiplication, division, and order of operations) • Expressions: writing and evaluation using substitution • Two-step equations with integer coeffcients, constants, and soluations.

NUMBER SENSE & COMPUTATIONAL FLUENCY

What are Learning Progressions?

Most children follow a natural developmental progression in learning mathematical concepts. A learning progression reveals the building blocks that students need to learn in order to develop the broader mathematical concepts. Critical concepts and skills learned in the early grades lay a foundation for deeper mathematics learning in the older grades.

Why Are They Important?

Knowing the key stages of learning progressions can help teachers assess their students' current understanding and plan for the next steps of instruction. This enables teachers to start where students are, not where we think they should be. If we build upon students' current knowledge and then move along a progression, we can decrease student anxiety and increase student success (Clements & Sarama, Learning and Teaching Early Math: The Learning Trajectories Approach, 2014).

How to Use Learning Progressions

Whole staff, resource teams, and teachers can use learning progressions to:

- Assess current understanding.
- Plan for instruction and scaffold learning.
- Differentiate instruction.
- Monitor student progress.
- Know that all the goals and activities we have planned are within the developmental capacities of children.

Please refer to the Balanced Numeracy Guide OneNote for links to videos and examples of learning progressions for other key concepts.

Learning Progressions in the Early Years

Counting

- Verbal counting
- Object counting
- Counting forward
- Counting backward
- 1-1 correspondence
- Cardinality
- Conservation
- Can name, write, and match numerals to numbers and quantities to 10

Subitizing

- Perceptual (can recognize quantities to 5)
- Conceptual (can group to determine quantity without counting)

Decomposing

- Counts on or back by groups to change a number
- Recognizes parts of a number to 5
- Recognizes parts within a larger number
- Recognizes parts within a larger number
- Identifies missing parts by + or – to 10

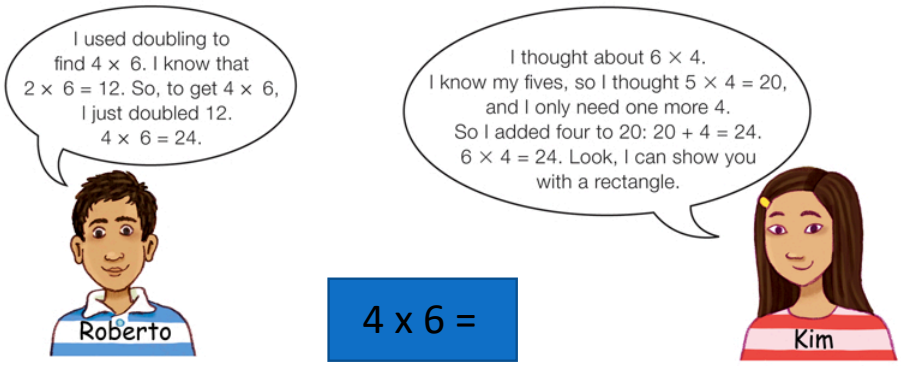
Place Value

- Composes teen numbers of tens and ones and decomposes teen numbers into tens and ones
- Bundles quantities into tens and ones
- Knows total instantly when the number of tens and ones is known
- Knows how many tens can be made by groups of ones and that the left over are ones knows 10 more and 10 less is able to use manipulatives to model a 2-digit written number

NUMBER SENSE & COMPUTATIONAL FLUENCY

Computational Fluency

Computational fluency is defined as having efficient, flexible, and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently (NCTM, 2000).“The computational methods that a student uses should be ***based on mathematical ideas that the student understands well***, including the structure of the base-ten number system, properties of multiplication and division, and number relationships” (Principles and Standards for School Mathematics, p. 152).



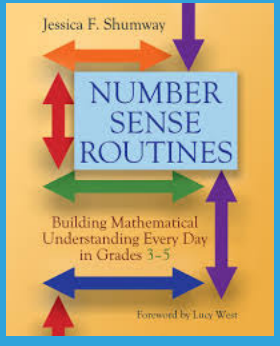
Developing computational fluency is an expectation of the BC Mathematics curriculum, and it is represented in the big ideas, curricular competencies and content in all grade levels. Computational fluency develops over time. Students usually develop computational fluency over three years. Students learn about the four operations, the properties of the operations, and strategies used to perform the operations. They apply this understanding to make computation more efficient.

How Can Computational Fluency Be Developed?

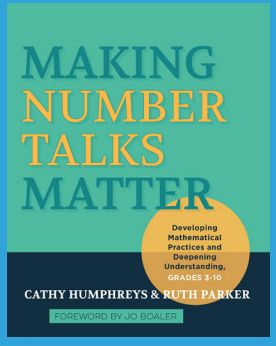
Fluency is about understanding, strategies, and efficiency—not merely speed. Conceptual understanding (number sense) and strategies are the foundations on which fluency is built. Teaching for both skill and understanding is crucial—these are learned together, not separately.

The following instructional routines are purposeful and powerful tools that can help students develop strategies, and mathematical discourse, and create awareness about compuuttional fluency.

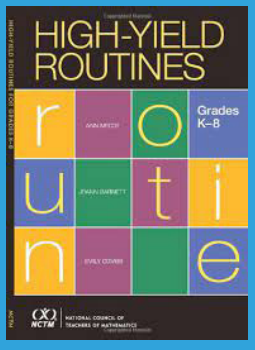
- Number/Fraction Talks
- Quick Images
- Today’s Number
- SPLAT!
- Math games
- Open number line
- Choral Count
- How do you know?
- Which One Doesn’t Belong?



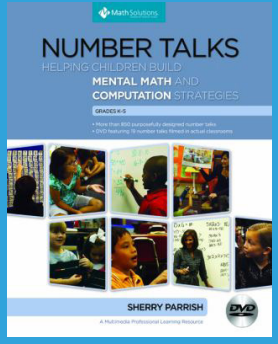
Number Sense Routines
Jessica F. Shumway



Making Number Talks Matter
Cathy Humphreys & Ruth Parker



High-Yield Routines
Ann McCoy, Joann Barnett, & Emily Combs



Number Talks: Helping Children Build Mental Math and Computation Strategies
Sherry Parrish

NUMBER SENSE & COMPUTATIONAL FLUENCY

How Can Computational Fluency Be Developed?

Teachers can help students develop flexible computational strategies in the following ways:

Number Talks

- Students are given a problem that involves addition, subtraction, multiplication, or division.
- The teacher encourages students to use a strategy that makes sense to them.
- Teacher facilitates discussion about the various strategies used to solve a problem.
- Students notice the effectiveness of the different strategies and begin to adopt them as their own.

Example: Number Talk Focusing on a Student's Strategy for Subtraction (Compensation)

A compensation strategy for subtraction involves subtracting more than is required and then adding back the extra amount.This strategy is effective when the number being subtracted is close to a friendly number (e.g., a multiple of 10). 565-283 is calculated by subtracting 300 from 565, and then adding back 15 (the difference between 285 and 300).

565 - 285
565 - 300 = 265
265 + 15 = 280

Number Strings

- The teacher writes the first computation horizontally on the board and asks students to calculate the answer.
- Students are given time to calculate mentally.
- The teacher asks a few students to explain how they determined the answer.
- The teacher models students' thinking on the board by using diagrams, such as open number lines, to illustrate various strategies.
- The teacher presents the remaining computations, one at a time. Strategies for each computation are discussed and modelled.

Example: Number String for Using the Compensation Strategy for Subtraction

56 - 30
56 - 29
56 - 28
344 - 200
344 - 199
344 - 197
546 - 196

SPATIAL REASONING

Early math skills and concepts go beyond numbers and counting. Spatial reasoning is another component of children's early math learning that plays an important role in their later math achievement. In mathematics education, it is believed that spatial reasoning is not a pre-determined ability but is something that can be developed in all children. Teachers can support their students' spatial thinking by providing experiences that help to develop the language and processes involved with spatial reasoning.

What is Spatial Reasoning?

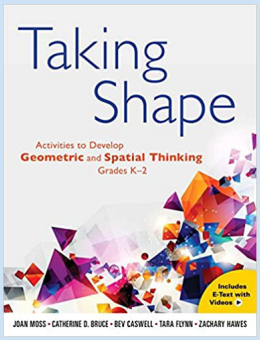
Spatial reasoning is the ability to create and manipulate mental representations of actual and imagined shapes, objects, and structures (Cohen & Hegarty, 2012). Put more simply, spatial reasoning is about being able to picture things in the mind's eye and to be able to mentally move, rotate, fold objects and shapes in space. Spatial reasoning is linked to geometry: shapes and spatial relationships.

Why is it Important to Teach Spatial Reasoning?

Spatial reasoning in young children can be an early indicator of their academic success, especially in numeracy. Spatial skills, such as visualizing spatial transformations, make it easier to think of numbers linearly, from smallest to largest, or to solve calculation problems mentally. Having the ability to mentally picture and manipulate objects can also help students become successful in science, technology, engineering, art, and math (STEAM). Spatial reasoning and geometry are foundational to disciplines such as astronomy, architecture, art, geography, biology, and geology, and are an essential part of future careers.

There is specific vocabulary or spatial language that needs to be developed in young children.

In their book, Taking Shape (2016), Canadian authors Joan Moss, Catherine D. Bruce, Bev Caswell, Tara Flynn, and Zachary Hawes describe five focus areas for spatial reasoning:



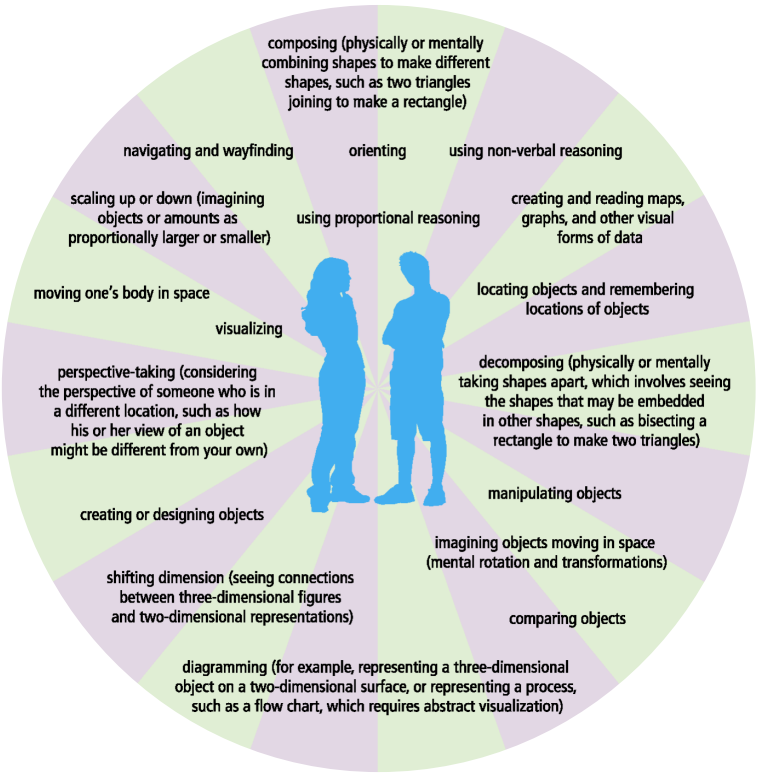
- **Symmetry**
  When one of two shapes are slid, flipped, or turned and matches the second shape exactly—two main types reflection (line) and rotational symmetry.
- **Composing and decomposing 2D images and 3D objects**
  Identifying shapes within shapes, composing a shape from two more smaller shapes.
- **Transforming**
  Predicting what a shape, image, or object will look like after it is flipped or rotated, conduct physical or mental transformations.
- **Locating, orienting, mapping, and coding**
  Placement or position of objects in space and understanding the relationships between those positions, sequences of numbers, and symbols to communicate an action.
- **Perspective-taking**
  The ability to picture in your mind a perspective other than your own (bug's eye view, bird's eye view).

SPATIAL REASONING

How Can We Teach Spatial Reasoning?

- Provide students with opportunities to notice, manipulate, and play with various blocks and materials. Have students describe the attributes of the materials such as colour, size, shape, and numbers of vertices, or straight or curved edges or sides. Also, focus on working spatially with the materials such as balance, movement, location, position, and orientation.
- Look at materials, story settings, and maps from different perspectives (bird's eye view and bug's eye view).
- Talk about space in the illustrations when you are reading to your students. "That building behind my house is really tall." Use gestures such as pointing or tracing objects to help students understand the spatial words you are using.
- When exploring and discussing 2D and 3D shapes, go beyond just labelling the shape. Focus on the shape's properties and use mathematical vocabulary to describe them. "While all rectangles have four sides, squares are a special kind of rectangle that have four sides that are all the same length."
- Have students explore a variety of both 2D and 3D shapes that can be manipulated, such as nets or skeletons of 3D shapes. This will help students to see and feel how 2D shapes "live" in 3D shapes.
- Introduce "spatial talk" or mathematical vocabulary to support students' thinking. Introduce terms that will support mathematical discourse when of interest to the students—terms such as parallel, intersecting, rotating, etc., might be interesting for students even though the vocabulary isn't at their grade level.

Spatial reasoning can involve:



http://www.edu.gov.on.ca/eng/literacynumeracy/Inspayingattention.pdf

"Spatial thinking is not an add-on to an already crowded school curriculum, but rather a missing link across that curriculum. Integration and infusion of spatial thinking can help to achieve existing curricular objectives."
(National Research Council, 2006, p. 7)

PROBLEM-SOLVING

Teaching Through Problem-Solving

Teaching through problem-solving plays an important role in the BC Mathematics curriculum. Students learn and understand mathematics through solving mathematically rich problems, and problem-solving skills are developed slowly over time. A problem-solving approach invites challenge and hard work, promotes students' conceptual understanding, fosters their ability to reason and communicate mathematically, and captures their interests and curiosity. There is no evidence that students sacrifice their basic skills if the focus is placed on developing their mathematical problem-solving skills. Rather, a problem-solving approach develops the core and curricular competencies of our BC curriculum.

Carefully planned problem-solving activities include:

- Finding multiple solution strategies for a given problem.
- Engaging in mathematical explorations and discourse.
- Giving reasons for their solutions.
- Making generalizations.

Marian Small, a Canadian mathematics professor and author, describes a good problem as a task that requires a student to think about math in a fresh way. In addition, she explains that good problems can have any of the following characteristics.

Good problems might:

- Require a student to connect previously learned ones.
- Be solved in many ways, often using different problem-solving strategies.
- Pique curiosity.
- Have personal meaning or relevance to the student.
- Have a creative element to it.
- Require reasoning.
- Often have many answers that bring about good discussion.

Marian Small, *Making Math Meaningful,* ***p. 98, 2017***

To be an effective problem solver, students need to be able to make sense of the math, apply a strategy that works for them, and notate or model their thinking. In the past, many teachers adopted the approach of teaching strategies systematically and in isolation (work backwards, find a pattern, draw a picture).

Cognitively Guided Instruction (CGI) approaches things differently. Students are presented with a problem in context and are invited to solve it any way they can. Strategies are then shared and discussed so that students can see that there are multiple ways to solve a problem as well as common strategies to solve a problem. As general strategies emerge, anchor charts and icons can be used to group solutions into common approaches. It is important to remember that *a strategy is what we do in our heads as we make sense of and solve problems*. Modelling or notating is how the strategy gets represented.

1. Students are given a problem or task to solve in context.
2. Students solve the problem any way they can.
3. Teacher observes and picks one or two students to share their strategies.
4. Students start to highlight and recognize common ways of approaching and solving problems. Students also observe unique strategies.

PROBLEM-SOLVING

What is a Task or Problem Worth Solving?

A problem worth solving is accessible to all students. It has multiple entry points, has a low floor (threshold) and a high ceiling. This means that these problems lend themselves to natural differentiation where all students can address the problem at their level and experience success. A problem worth solving allows the use of multiple strategies and may or may not have more than one solution.

Rich Math Tasks/Problems

Rich mathematical tasks usually have the following characteristics:

- Accessibility to all learners (low threshold, high ceiling)
- Multiple approaches and representation
- Fixed constraints (to avoid the overwhelming number of possibilities)
- Collaboration and discussion
- Engagement, curiosity, and creativity
- Opportunities for extension

Examples of Traditional Problem versus Rich Math Tasks/Problems

Traditional Task/Problem	Rich or Open Taks/Problem
Calculate -2 + -1 =	Two numbers added together create a sum of -3. What might the numbers be?
Skip count by 2's to 50 Skip count by 5's to 50	You have 42 grapes in a bowl. If you dumped them out, how could you organize them into equal groups to make it easier for a friend to count? Show your work.

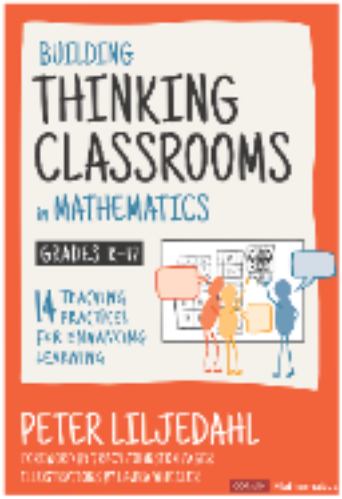
Below are some of the websites of rich tasks/problems that are widely used by teachers.

- **Openmiddle.com**
  K-12 challenging math problems.
- **tapintoteenminds.com/3act-math/**
  K-12, Three Act Math, real world math problems.
- **www.cemc.uwaterloo.ca/resources/potw.php**
  Grades 3-12, problem of the week.
- **peterliljedahl.com/teachers/numeracy-tasks**
  K-12 collection of numeracy tasks co-constructed with Peter Liljedahl.
- **youcubed.org/tasks/**
  Hosted by Jo Boaler, tasks for K-12.
- **nctm.org/Classroom-Resources/CRCC/Problems-of-the-Week-Resources/**
  NCTM Problem of the Week (membership required).
- **gfletchy.com/3-act-lessons/**
  Math tasks that consist of three parts: a provocation, a video, and the reveal.
- **estimation180.com**
  Estimation challenges.
- **bstockus.wordpress.com/numberless-word-problems/**
  Numberless word problems to provide scaffolding.

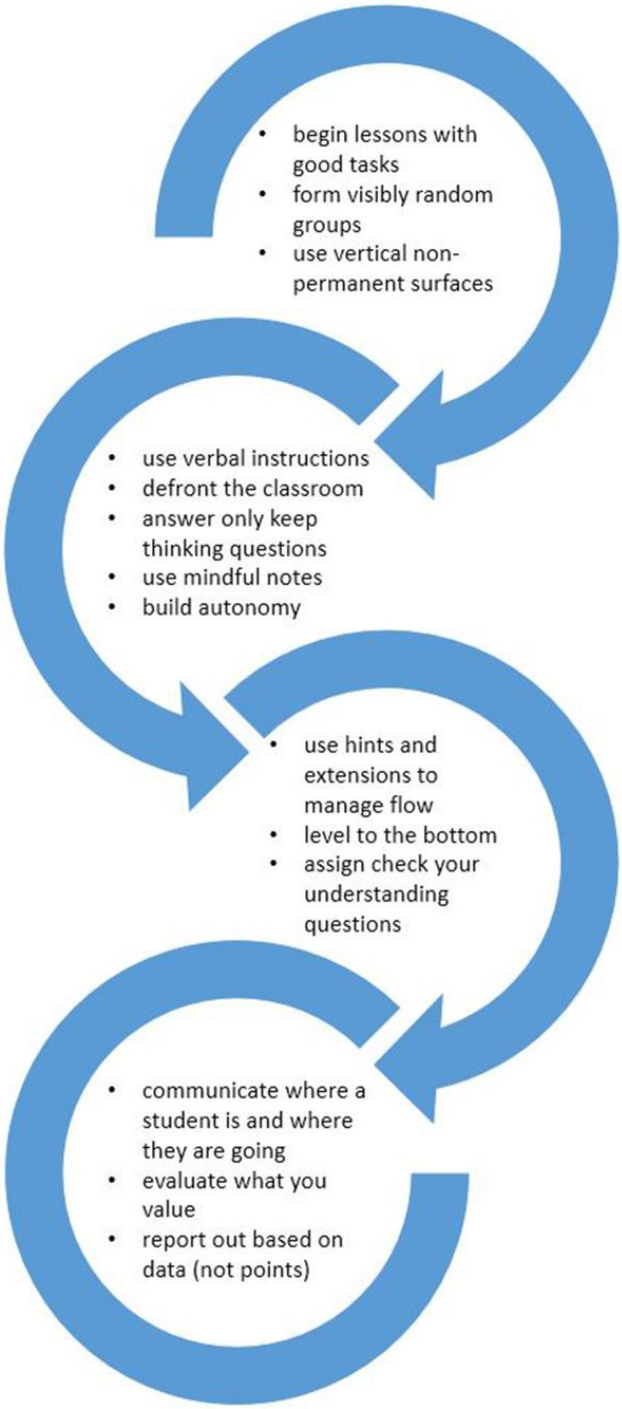
PROBLEM-SOLVING

Building a Thinking Classroom in Mathematics

To foster and develop effective problem-solving in the classroom, Peter Liljedahl encourages teachers to cultivate a "Thinking Classroom." The Thinking Classroom is a space where students think individually, think collectively, learn together, and build understanding through activity and discussion. Working through rich numeracy tasks encourages students to use the numeracy thinking processes to interpret, apply, analyze, and communicate their solutions.



Liljedahl, author of **Building a Thinking Classroom in Mathematics,** has developed a 14-point plan for encouraging students to engage deeply with math content:



Types of Numeracy Tasks

A task that requires students to use mathematics and apply curricular/core competencies in a real context or situation is usually considered a Numeracy Task.

There are four types of Numeracy Tasks, all which promote a Thinking Classroom.

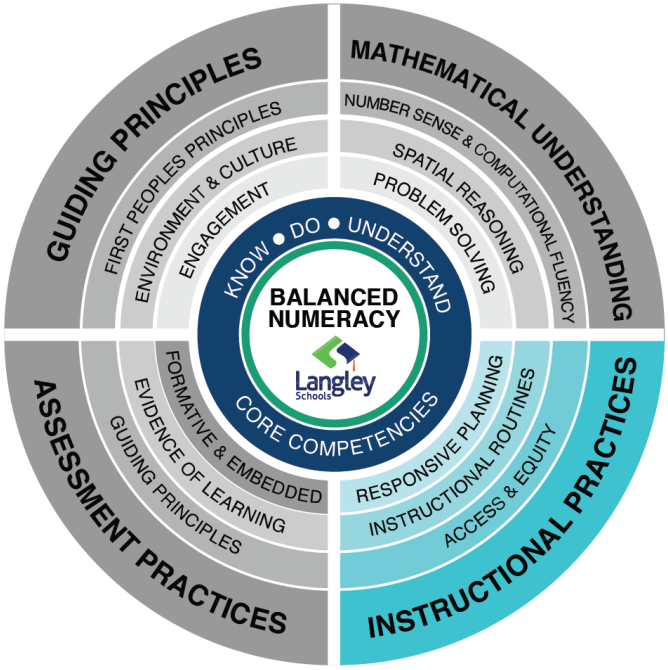
- **Reasoned Estimates**
  Require estimations in order to build a logical argument and arrive at a possible solution. (e.g., travelling to Australia).
- **Plan and Design**
  May require students to analyze time, space, cost, and people in order to make a recommendation (e.g., shipping several containers).
- **Fair Share**
  students need to decide how to best share something fairly (e.g., giving out bonuses).
- **Model**
  Students need to produce a model or strategy, given a data set; and then to apply this model or strategy to a new data set and, if necessary, to refine the model (e.g., ranking criteria).

Instructional Practices

Is it important to have a diverse repertoire of instructional practices to uncover the curriculum, build conceptual understanding, and allow for equitable access to learning. Considerations for planning, routines that support mathematical thinking, and inclusive practices are described in this section of the guide.

Access & Equity

ALL students should be provided with access to high quality numeracy experiences and high expectations for mathematics learning (NCTM position). Teachers should consider students' knowledge, backgrounds, experiences, and cultural perspectives when designing their mathematics program.



Instructional Routines

Instructional routines are structured activities that can help foster the core/curricular competencies in relation to mathematical content. Implementing instructional routines provides access for all learners and can help to build a community of learners through discussion, questioning, and risk taking.

Responsive Planning

Teachers plan the instruction and learning so that the various needs of all students are addressed and so that students can see themselves reflected in classroom resources and activities. When planning, it is important to consider:

- What students need to learn.
- How we will know whether students have learned it?
- How can we plan instruction to promote the learning?
- How will we respond to students who need more support or require an extension?

This section highlights key strategies to consider as teachers plan effective and inclusive programs for all students, whether daily, weekly, or yearly.

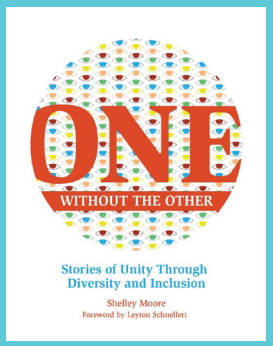
ACCESS & EQUITY

"All students should have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential."

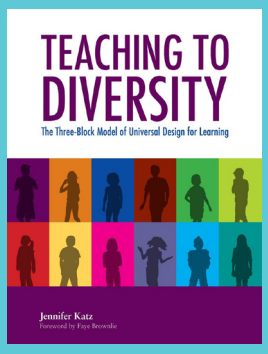
NCTM, Principles to Actions: Ensuring Mathematical Success for All, 2014.

Considerations

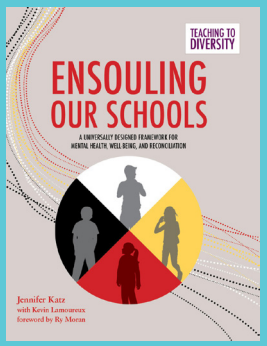
- Provide ALL students with access to high quality numeracy experiences and high expectations for mathematics learning.
- Be responsive to students' backgrounds, experiences, cultural perspectives, traditions, and knowledge when designing and implementing a mathematics program.
- Universal Design for Learning provides a planning structure for teachers to help support the needs of all students.
- Provide high level content to ALL students through parallel tasks where the same learning standard is being addressed.
- Encourage multiple ways for students to demonstrate their understanding.
- Support a positive "math identity" development through strengths-based teaching and learning practices.
- Provide opportunities for student voice and choice.
- Plan for inclusive practices, such as accommodations for written output and sensory/physical needs.
- Implement flexible groupings whenever possible:
  - Whole group: instruction and community building.
  - Small group: guided math, differentiation, exploration.
  - Individual: conferencing, interviews, explicit instruction, authentic practice.
  - "Visibly random groupings".



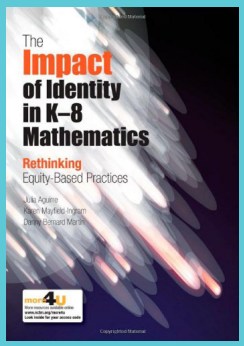
One Without the Other
Shelley Moore



Teaching to Diversity
Jennifer Katz



Ensouling Our Schools
Jennifer Katz



Impact of Identity in K-8 Mathematics: Rethinking Equity-Based Practices
Julia Aguirre, Karen Mayfield-Ingram, & Danny Martin

ACCESS & EQUITY

Technology

When used effectively, technology can enhance the teaching and learning of mathematics by:

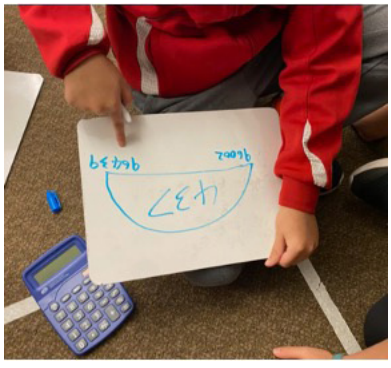
- Creating opportunities to explore mathematical ideas in multiple ways.
- Providing multiple forms of communication and ways to capture and explain learning and thinking.
- Bringing mathematical ideas to life through visual representations.
- Fostering collaboration and mathematical discourse.
- Providing access points for equity and diversity.

For technology to be integrated in a way that is pedagogically appropriate for math instruction, teachers need to consider:

- The technology knowledge the students will need.
- The mathematical understanding they'll need.
- The best practices for teaching both the technology and the math.

> "Mathematics is a subject that allows for precise thinking, but when that precise thinking is combined with creativity, flexibility, and multiplicity of ideas, the mathematics comes alive for people. Teachers can create such mathematical excitement in classrooms, with any task, by asking students for the different ways they see and can solve tasks and by encouraging discussion of different ways of seeing problems."
>
> ***(Boaler, Mathematical Mindsets, 2015, p 59)***

The Mission of the Langley Technology Action Plan for Learning is "to inspire, support, enhance and transform learning using technology; empowering all learners to be innovative and reach their full potential".



ACCESS & EQUITY

The Role of Technology in the BC Mathematics Curriculum

How Can Technology Enhance the Teaching and Learning of Mathematics?

This is the language for the K-5 and Grades 6-9 Curricular Competencies that focus on the use of technology in the math class.

GRADES K-5

Reasoning & Analyzing

- Use reasoning to explore and make connections.
- Estimate reasonably.
- Develop mental math strategies and abilities to make sense of quantities.
- Use technology to explore mathematics (calculators, virtual manipulatives, concept-based app).
- Model mathematics in contectualized experiences.

GRADES 6-8

Reasoning & Analyzing

- Use logic and patterns to solve puzzles and play games including coding.
- Use reasoning and logic to explore, analyze, and apply mathematical ideas.
- Estimate reasonably.
- Demonstrate and apply mental math stategies.
- Use tools or technology to explore and create patterns and relationships, and test conjectures (acting it out, using manipulatives, drawing diagrams, building, programming).
- Model mathematics in contextualized experiences.

GRADES K-8

Communicating & Representing

- Use mathematical vocabulary and language to contribute to mathematical discussions.
- Explain and justify mathmatical ideas and decisions.
- Communicate mathematical thinking in many ways (may use technology such as screencasting apps, digital photos).
- Represent mathematical ideas in concrete, pictorial, and symbolic forms.

There are many applications that allow students to visualize and experience mathematics in ways they would not be able to without the technology. Desmos and Gizmos are two examples of applications that can transform teaching and learning in mathematics. Another aspect of using technology in the math classroom is to help students represent and share their learning, as in screencasting or recording apps. Students who need support with communication can take advantage of the accessibility features available on most devices. Technology can also increase student engagement by providing choice and differentiation.

Using Math Apps

There are many apps that can support mathematics learning—some are mathematics specific and others are used to represent and share learning. Avoid the type of math apps that are similar to a worksheet or a timed math drill. Choose apps that provide student feedback, visual supports and encourage problem-solving or mathematical thinking.

Author Tracy Zagar uses a baseline set of criteria when choosing math apps by asking three questions:

- **Is there time pressure?** Timed learning conditions can cause math anxiety.
- **Is it conceptual?** Apps that provide models such as arrays or number lines can build understanding better than the flash card types.
- **How does it handle mistakes?** Students need to figure out why they made errors rather than just moving to the next problem after a mistake.

App Approval Process

The Langley School District has implemented a process for requesting iPad apps. If you wish to have an app included on district iPads, follow the procedures that are described in this linked document:

https://langleyschoolsca.sharepoint.com/:p:/s/SBOAppTeam/Eb93k-LJZoVFgsqNaRCZPmwB5MNgJU2AICTCvmBr0oBw2w?e=jNP9l5

INSTRUCTIONAL ROUTINES

What is an Instructional Routine?

An instructional routine is a familiar structure with an open-ended task where less time is spent on teachers providing directions and more time is spent on the learning of mathematics. A good routine provides all students opportunities to do mathematics and gives teachers insight into student thinking. Instructional routines should be intentionally planned to move learning forward in response to where students are in their mathematical learning:

- Start with an open-ended prompting question that allows every student access to the mathematical discussion.
- Provide time for students to think, share and reflect.
- Create agreed-upon expectations of how students share their thinking.
- Respect one another to build a mathematical community.

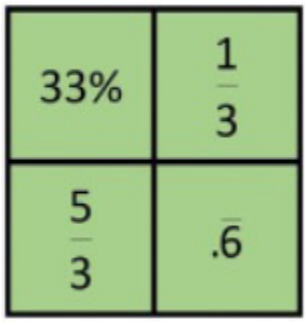
> "As teachers from multiple grade levels plan and consistently implement common routines, students will experience a more coherent and better-articulated mathematical experience. In addition, the use of common models such as number lines and Venn diagrams within the routines will improve students' ability to accurately and strategically use these models."
>
> ***Barnett, Combs, McCoy, High-Yield Routines Grades K-8, 2013, p 3***

Why use Instructional Routines?

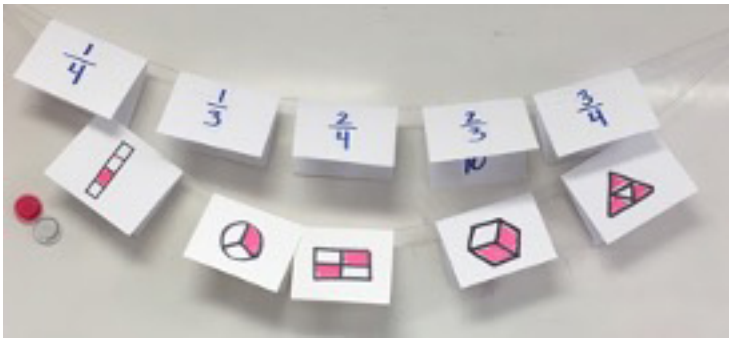
Instructional routines are used to support student thinking and learning in many ways, including:

- Connecting curricular competencies with content to understand the big ideas.
- Building conceptual understanding by exploring, problem-solving, reflecting, connecting, communicating, applying, and collaborating.
- Playing with mathematical ideas.
- Encouraging multiple ways for students to demonstrate their understanding.
- Creating opportunities for mathematical discourse and student voice.

Math Routine: Which One Doesn't Belong?



Math Routine: Clothesline Math



INSTRUCTIONAL ROUTINES

The following is a list of instructional math routines and their websites.

Routine and Website	Description
Number Talks tedd.org/number-talks/	Students are given a mental math problem and share strategies for solving the problem aloud in a group setting as the teacher documents student strategies.
Number Talk Images ntimages.weebly.com	A collection of images to be used as a launching point for Number Talks.
Would You Rather? wouldyourathermath.com	Provides students with two options, both of which require basic computation, comparing value, etc.
Which One Doesn't Belong? wodb.ca	Find a reason for why each one doesn't belong and compare ideas.
Splat! stevewyborney.com	K-12 interactive number sense strategy that can be used at any grade level.
Fraction Talks fractiontalks.com	Simple visuals to foster creative thinking around fractions.
Agree or Disagree aodmath.com	A provocation, such as a number or shape, of which students 'construct arguments'.
Same or Different samedifferentimages.wordpress.com	Students construct arguments when comparing objects, such as numbers or shapes.
Clothesline Math Clotheslinemath.com	Students consider and discuss how to place and space numbers in relation to each other on an open number line.
Counting Routines tedd.org/mathematics/	Counting collections, choral counting, Quick Images etc. help students develop skip counting and counting in groups. You will need to register to access the website (free).
Visual Patterns visualpatterns.org	Hundreds of pattern images to help students sharpen their algebraic reasoning and critical thinking.
Estimation 180 estimation180.com	Estimation tasks that make mathematical reasoning accessible to students and enjoyable.
Esti-Mysteries stevewyborney.com/category/esti-mysteries/	
High Yield Routines blogs.sd38.bc.ca/sd38mathandscience/wp-content/uploads/sites/14/2021/10/SD38_High-Yield_Routines_September-2015.pdf	Seven math routines described in the book, High Yield Routines.

INSTRUCTIONAL ROUTINES

Teaching Math Games

When considering using games for teaching mathematics, educators should distinguish between an 'activity' and a 'game'. Gough (1999) states that "a 'game' needs to have two or more players, who take turns, each competing to achieve a 'winning' situation of some kind, each able to exercise some choice about how to move at any time through the playing". The key idea in this statement is that of 'choice'. In this sense, something like Snakes and Ladders is NOT a game because winning relies totally on chance. The players make no decisions, nor do they have to think further than counting. There is also no interaction between players—nothing that one player does affects other players' turns in any way.

Games in the Math Program

- Playing games can deepen conceptual understanding, increase learning, test intuitive ideas, and encourage problem-solving, and strategic math thinking.
- Games can provide meaningful situations needed to develop fluency. They are motivating, engaging, novel, and fun. They encourage positive social interactions and can boost self-esteem.
- Games allow for differentiation and multiple skill levels and allow students to learn from one another.
- Strategy games offer far better results in improving computational fluency, efficiency and accuracy than drill techniques and timed tests.
- Games offer teachers a fantastic means of assessment of learning. Playing a game with a child allows their thinking to become apparent in a much less threatening environment.
- Games encourage independence.
- Games have few language barriers as many structures and procedures to games are common to different cultures.
- Structuring games in the classroom frees teachers up to meet with other kids, provide remedial support, assess for learning, or provide challenges as needed.

To access math games visit the TCS Team SS-Grade 3 and Grades 4-8, Numeracy channel > Files.

People of all ages love to play GAMES that are fun and motivating. Games give students opportunities to explore fundamental number concepts, such as the counting sequence, one-to-one correspondence, and computation strategies. Engaging mathematical games can also encourage students to explore number combinations, place value, patterns, and other important mathematical concepts. Further, they afford opportunities for students to deepen their mathematical understanding and reasoning. Teachers should provide repeated opportunities for students to play games, then let the mathematical ideas emerge as students notice new patterns, relationships, and strategies.

Kitty Rutherford, Posted April 27, 2015 nctm.org

Oldfield (1991) says that mathematical games are 'activities' which:

- Involve a challenge, usually against one or more opponents.
- Are governed by a set of rules and have a clear underlying structure .
- Normally have a distinct finishing point that has specific mathematical cognitive objectives.

Evergreen Games

Evergreen games are games that have general rules that never change. Once you teach children those rules you can use the game for every math concept. For example, the rules of Memory never change... but what matches they are looking for can change with each new concept.

Examples of evergreen games are:

- Bump
- Memory
- I Have/Who Has
- Capture 4
- Difference To...

These examples are readily available, or you can make them with your students using a blank template.

RESPONSIVE PLANNING

Structures & Strategies

Planning is the key to effective math instruction. To plan for effective instruction, teachers need to consider what they want students to learn, how they will know whether students have learned it, how they will design instruction to promote the learning, and how they will respond to students who are not making progress or need their learning extended. Plans should include both long-term (yearly, monthly) and short-term (lessons) to ensure that the learning standards and topics are addressed.

There is great opportunity to build a strong mathematical foundation by intentionally planning for and revisiting mathematical concepts and experiences throughout the school year. By spiralling the learning and the exploration of concepts, students continually deepen their understanding and hook new learning onto greater context and experience. Exploring mathematical concepts over time instead of in isolation allows students to build connections and deepen their conceptual understanding.

When Planning, Ask Yourself:

- In what ways does this concept connect with the students' experiences in and out of school?
- How can I provide opportunities to practice understandings using concrete materials?
- How will students show what they know?
- How will I understand what they still need to know?
- How can I develop further mathematical connections?
- How can I hook this mathematical learning onto other mathematical concepts?
- In what ways can I revisit this concept in other subject areas?
- What materials will help facilitate understanding?
- How can students apply their understanding to real world situations?
- How can I plan in ways that are culturally responsive? Plans which recognize that all students learn in ways that are connected to background, language, family structure, and social or cultural identity.
- How can I create opportunities for students to have ongoing practice with concepts?
- Where can I plan for new learning, practice, and revision?

Plan for Concepts to be Spiraled Throughout the Year

- Daily plans include direct teaching or practice of concepts.
- Weekly plans include the development of math through whole group lessons, small group lessons, or practicing of skills.
- Monthly plans weave real life examples, links to previous learning, and time for deep understanding through hands-on practice and application.
- Yearly plans include finding opportunities to spiral previous learning rather than teaching concepts in isolation without revisiting.

The BC Curriculum provides great opportunity for skills to be interwoven through different subjects and exploration of the core competencies.

- Math intersects with all other subjects. Our lives are interdisciplinary and the more we replicate that using traditionally siloed subjects the more our students will be able to relate.
- Designing learning experiences that pair math with other subjects allows greater time for exploration of concepts.
- An inquiry approach can be used to integrate the various subject areas. Designing these learning experiences with co-workers can be powerful.

RESPONSIVE PLANNING

Different Lesson Styles

There are various types of lesson structures that teachers can use for math instruction. It is important for teachers to vary the type of lesson style to meet the different learning needs of students. In addition, a specific type of lesson might better suit the mathematics that needs to be taught or be more suitable for the purpose of the lesson. It is recommended that teachers try to create a balance of the types of lessons that they will use, as explained below.

Exploration/Inquiry/Problem Based

Students may use different approaches and produce different conclusions. *Example: Open Questions. "How many different ways can you make 24?"*

Direct Instruction

Students are taught a mathematical skill or concept under the direction of the teacher. *Example: Students learn how to read both an analogue and digital clock.*

Guided Activity/Guided Practice

Students show what they know and demonstrate the skills and concepts they are learning with teacher support. Students practice skills on their own for the first time, teacher gives feedback to all and provides support for learners that need it. *Example: Students build arrays to show the different factors for 48.*

Mini Lesson

Brief, 10-15 minutes, includes only one focus or teaching point. *Example: Students and teacher co-create an anchor chart for classifying angles.*

Components of a Lesson

Most lessons, depending on the type, have a three-part structure: before, during and after. Each of these parts serve a specific purpose. The first part of a lesson usually helps the student connect to the topic. A "hook", inquiry question, or warm-up activity are examples of strategies teachers might use to engage students. During the second part of the lesson, students spend most of the time working on a problem or activity, individually, in pairs, or small groups. The last part of a lesson is a consolidation to share and discuss strategies or debrief the essential understandings of the lesson. It is worthwhile for students to be actively involved and have opportunities for discussion in all three parts of a lesson.

Example of a Three-Phase Lesson Structure for Problem-solving (John Van de Walle)

BEFORE	DURING	AFTER
Getting Ready • Activate students' prior knowledge. • Pose a thought provoking question. • Discuss and clarify task; students ask questions. • Establish clear expectations.	**Working on It** • Let's go! • Students explore and develop strategies. • Notice students' mathematical thinking, observe, and assess. • Encourage students to clarify ideas. • Provide appropriate support and extensions.	**Consolodation** • Class discussion: promote a mathematical community of learners. • Bring students together to share, analyze and justify strategies and solutions. • Ensure concepts are drawn out of problem. • Clarify misconceptions. • Summarize discussion and emphasize key points.

RESPONSIVE PLANNING

Universal Design for Learning (UDL)

All students can succeed in learning mathematics. But in most classrooms, students may demonstrate a wide range of strengths and needs. UDL is an approach that enables teachers to provide personalized, precise teaching and learning experiences for all students. The goal of UDL is to create a learning environment that is open and accessible to all students. Instruction based on principles of universal design is flexible and supportive, can be adjusted to meet different student needs, and enables all students to access the curriculum as fully as possible.

1 Define flexible, clear SMART goals.
Ask yourself, "What is the goal of this lesson?"

2 Consider learner variability.
Anticipate challenges for students or other areas of need. Apply UDL guidelines to help plan for differentiation and determine what scaffolds are needed (see chart).

3 Determine appropriate assessments.
Choose assessments that are flexible and assess individual student growth.

4 Select methods, materials, and media.
Determine which strategies and materials can address the different needs of students.

5 Teach and assess learning.
Use flexible and formative assessment tools that connect to the learning goal.

6 Self-reflection.
Review the observations and assessments. Did students meet the learning goal? What went well or need to change for next time?

Know your learners / Plan for predictable variability

Design from the edges

Seek diverse perspectives

Learning Environments

Universal Supports

Ask: Does the everyday learning environment provide a range of supports to cater for learner variability?

Curriculum and Lesson Planning with UDL

Goal or Purpose

Ask: Does everyone know what they are doing and why?

Provide multiple means of **Engagement.**
Purposeful, motivated learners.

Provide multiple means of **Representation.**
Resourceful, knowledgeable learners.

Provide multiple means of **Action & Expression.**
Strategic, goal-directed learners.

Methods or Activities

Ask: Will the teaching methods and options for completing activities work for everyone?

Resources & Materials

Ask: What resources or materials will I need to make it work for everyone?

Assessment and Outputs

Ask: Can the students show what they know in a way that works for them?

Itentional design

RESPONSIVE PLANNING

Structures & Strategies

Planning for instruction includes creating responsive and flexible groups where students are able to participate in mathematical conversations that push their thinking.

Whole Group

This grouping is used for instruction and to build community understanding. Whole group instruction provides teachers with a quick method (mini lesson) of presenting information to all students.

Small Group

Guided math allows teachers to differentiate and target instruction for small groups based on proficiency levels. These groups are fluid as proficiency changes based on ongoing, formative assessment. Using visibly random groupings helps students move their knowledge throughout the classroom and creates more independent students that do not rely as much on the teacher (Liljedahl, 2016).

Independent

As students work independently on authentic practice, the teacher is conferencing with the students about their mathematical understanding and providing explicit instruction to those that need it.

Whole Group Lessons

- Presenting mini-lessons.
- Set in motion mathematical strategies.
- Read aloud math literature.
- Practice and review.
- Introducing the Math Workshop aspects for the week.

Small Group Lessons

- Differentiating instruction.
- Teaching with manipulatives.
- Teaching concepts often found to be difficult year after year for students.
- Assessing student learning informally.
- Supporting and practicing process (rather than content) skills like problem-solving, communicating, making mathematical connections, mathematical reasoning, etc.
- Support learning by giving feedback.

Independent

- Review previously mastered concepts.
- Practice math fact fluency.
- Math games to reinforce concepts.
- Practice problem-solving.
- Write in math journals.
- Complete computer-related work.
- Complete math-related work from other subjects.
- Complete work from small group instruction.

RESPONSIVE PLANNING

Math Workshop

Math workshop is a model of instruction that allows all students to be engaged. Supporting all levels of student thinking is purposeful and plentiful and is at the center of the entire philosophy of math workshop (Lempp, 2017).

Math workshop is designed to incorporate whole class, small group, and independent work in a community of learners that are developing a mathematical mindset.

You may have seen these words used interchangeably—math workshop and guided math. Math workshop is the overall model where the warm-up, mini-lesson, guided practice, independent practice, and closing occurs. Guided math is a component of math workshop where the teacher meets with students in small groups and differentiates the math to the needs of the students.

Why Math Workshop?

- Allows for differentiation and flexibility in meeting individual student needs.
- Incorporates daily spiral review and multiple exposure to concepts.
- Provides an opportunity for questions and immediate feedback.
- Promotes problem-solving, collaboration, cooperation and group social skills.
- Allows for ongoing formative assessment to guide instruction.

What it is

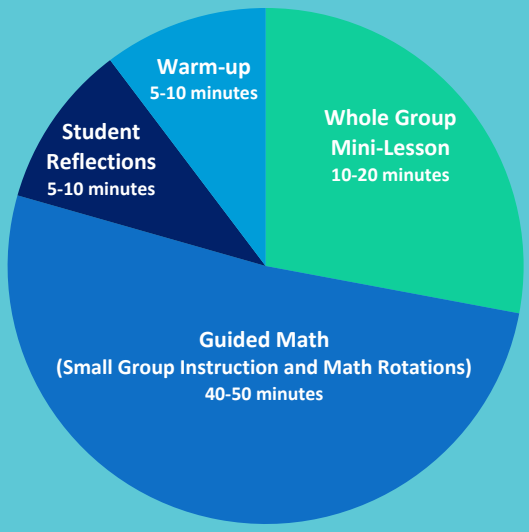
- Students doing most of the math.
- Student choice.
- Students talking about their mathematical thinking and reasoning.
- Students struggling with challenging mathematics and learning from errors.
- Teacher working with small groups.
- Focused on conceptual understanding.

What it is Not

- Teachers doing most of the math.
- One assigned worksheet.
- Teachers showing the procedure and talking about the steps to follow.
- Teachers rescuing students.
- Teachers presenting to the whole class.
- Focused on procedural skill.

Math Workshop Model

- **Math Warm-up (Number sense routine/ anchor task).** The number sense routine is like a math warm up for the brain. (See the section on routines for more information.)
- **Whole Class Focus Lesson.** Serves to build community understanding of a concept. The lesson is short and targeted.
- **Workstations and Guided Math Groups**. The workstations are differentiated, standards-based, engaging activities that foster mathematical thinking (Newton, 2016).
- **Debrief or Student Reflection**. Students engage in conversations about how they think mathematically, how they solved problems and how they made mathematical connections. Students ask questions, clarify their thinking, modify their work, and add to their collection of strategies. This step is very important and should not be skipped. If you run short on time, adjust your time within the work-stations. Reflections can be recorded in a math journal.



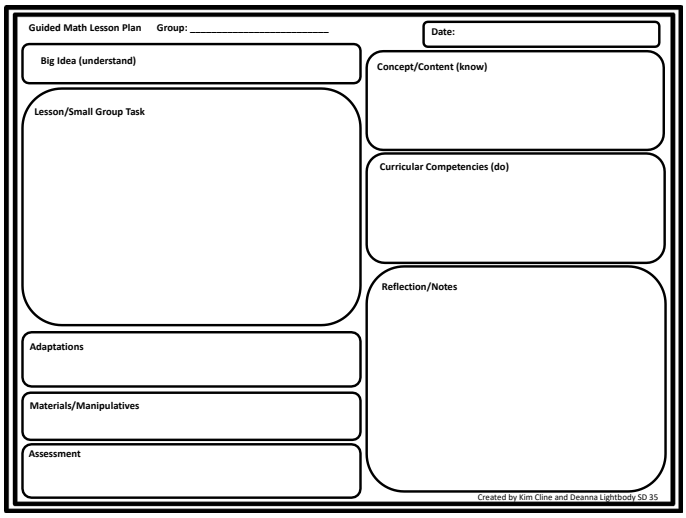
RESPONSIVE PLANNING

Guided Math

Guided Math is much like Guided Reading in that it is centred on working with a small group and focuses on concepts or strategies that may have been taught in a whole-class setting. In the small group, you will have the attention of fewer students, and so you are much more able to analyze their thinking. This way, you can respond directly to what you are seeing and hearing (Lemmp, 2017).

What Do I Teach During a Guided Math Lesson?

- Re-teach or reinforce a math concept
- Problem-solving
- Introduce a new workstation
- Observe/assess students
- Discuss common errors
- Provide intervention

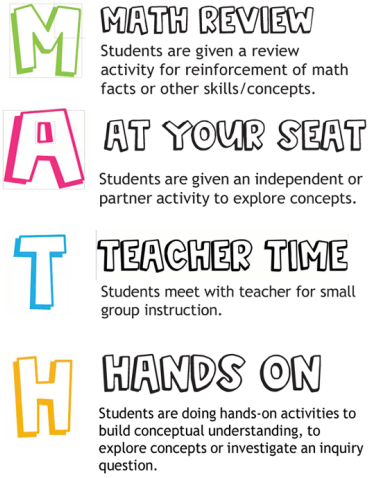


What Are the Rest of the Students Doing During Guided Math?

Math workstations allow students to engage in meaningful math practice or explorations while the teacher is working with small groups. Workstations are tasks where students often work in pairs or small groups but may work alone. The tasks or activities included in the workstations should allow students to gain greater mathematical proficiency and confidence in their mathematical abilities independent of the teacher. Procedures and expectations should be practiced beforehand to ensure that students can work independently to allow the teacher uninterrupted teaching time.

Math Workstation Examples

- Follow-up work from small group lesson
- Problem-solving
- Math games (fluency)
- Exploration with manipulatives
- Writing about math (math vocabulary, math journal prompts, inquiry)
- Multidisciplinary connections (where language arts, science, or social studies relate to math).
- Children's literature
- Technology



RESPONSIVE PLANNING

Home to School Connections

The home to school connection can play a significant role in achievement of students in mathematics. There are three key components of the home - school connection that can affect students' attitude towards math and their success in the classroom. The teacher's relationship with families, the type of parent support, and the type or amount of homework can directly influence students' progress in numeracy.

Relationships with Families

Many parents learned math through more traditional methods whether it was at school or at home. It is important for teachers to connect with parents early in the year, to help them understand the approaches that will be used for teaching mathematics and the reasons for these strategies/approaches. Teachers can help parents understand how their children learn mathematics in ways that are friendly, free of jargon, and informative.

- Plan family games nights where students and parents play math games taught in class.
- Create short videos or take pictures so parents can view their child performing a math task.
- Send regular emails or newsletters to update parents about the math concepts taught in class and ways to support at home.
- Share websites that can provide extra practice at home that is meaningful and engaging.

Support at Home

Parents want their children to be successful in school and often try to find ways to support them at home. It is important for teachers to help parents understand that the classroom teacher will provide the instruction, and parents are encouraged to provide real-life numeracy experiences that enhance the concepts taught in the classroom. Parents are the first influence in a child's life about their feelings about math, so it is crucial that parents remain positive when working with their child. The following are some general ways that parents can make math meaningful for their children and provide extra practice at home.

Counting
Use cupcake baking trays, egg cartons, etc., to count anything. Parents can watch to see how their child chooses to sort and count and say, "Tell me about how you counted." "How many?" Older students can count groups of objects to practise skip counting and multiplication.

Board Games
Younger students can play board games that require counting and skip counting as they move places along the board. Other games such as Cribbage, Monopoly, Yahtzee, and Prime Climb require additional math skills and strategic thinking.

Cooking
Children can learn all about measurements as they learn to cook or bake; older children may have to covert measurements. Students can also learn about fair shares and fractions, and record/compare nutritional values of recipes.

Noticing Math
The more parents can point out math in the home environment and community, the more children can make connections to math and build number relationships within math concepts. (e.g. We say "quarter past 4" when telling time, and a child may connect that to fractions: ¼ and money: 4 quarters = $1.00 with prompting and support from parents.)

Assessment Practices

Effective classroom assessment practices support student achievement by informing students, parents, and teachers on where students are at with their learning and establishing new learning goals (BC Curriculum, 2017). Research indicates that there are common principles that can be used to determine quality assessment. This section describes these principles of assessment, ways teachers can gather authentic information, and how to embed formative assessment in the classroom. For more information about assessment, see the Langley document, "CSL and Rethinking Assessment Guide."

Guiding Principles

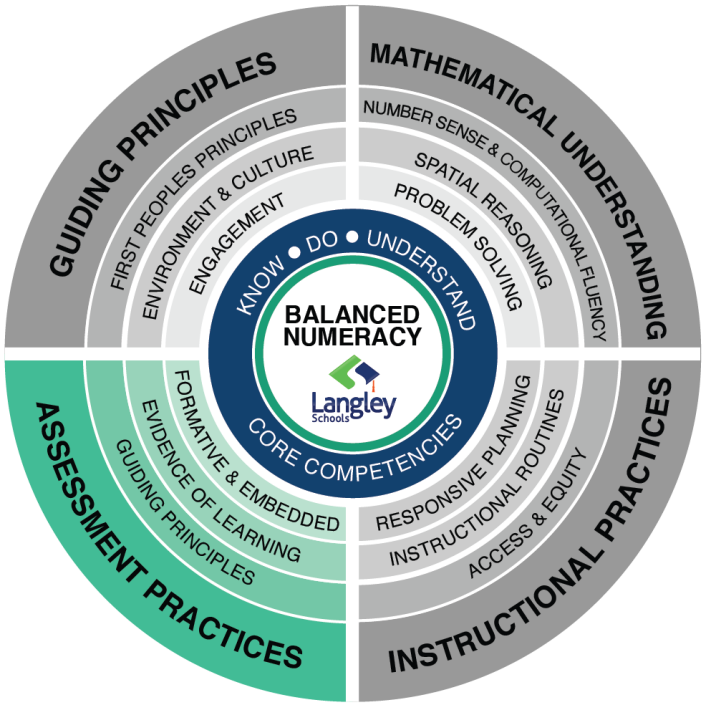
There are fundamental practices to consider when designing and implementing effective assessments in the math classroom. These guiding principles are based on evidence-based research and support a balanced approach to quality assessment.

Evidence of Learning

All assessment is designed with the student in mind and is strength-based. Teachers are encouraged to obtain information from a variety of sources, such as products, observations, and conversations with the students. At times, teachers can provide students with a choice to show their mathematical understanding in multiple ways to accommodate different learning styles and strengths.

Formative and Embedded

Formative assessment informs both the teacher and the student of the learning that is happening in that moment of time. Embedding assessment into daily classroom experiences provides opportunities for students to demonstrate what they know, understand, and can do in relation to the Mathematics learning standards.

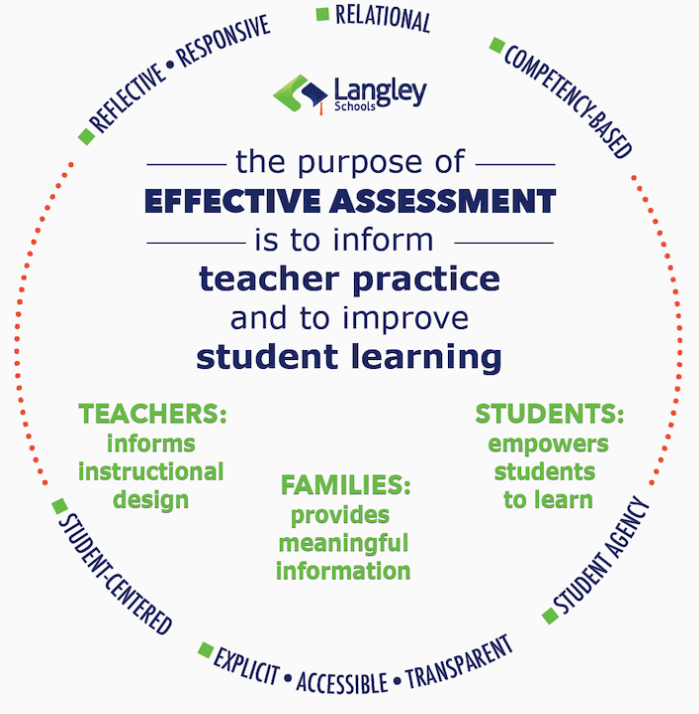


"The primary purpose of assessment is to improve student learning."
Anne Davies, PhD, Making Classroom Assessment Work, 2011.

"Minute by minute, day by day, strategic formative assessment can substantially improve student achievement."
Dylan Wiliam

GUIDING PRINCIPLES

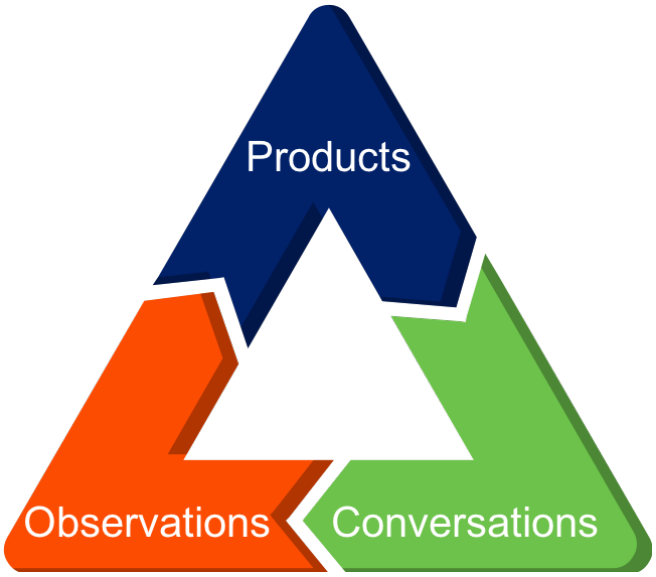
Effective Assessment



Competency-Based	Effective assessment reflects where the student's skill level is in relation to the curricular competencies. It balances the measurement of both mathematical content and processes (conversations, observations, and products). Evidence of learning and professional judgement determines proficiency.
Explicit, Accessbile, and Transparent	Effective assessment requires clear learning intentions and communicates success criteria so that learning is accessible for all students.
Relational	Effective assessment fosters hope, efficacy, and a culture of learning.
Reflective and Responsive	Effective assessment includes ongoing descriptive feedback to promote student reflections and inform next steps. It focuses on student improvement and measures growth over time.
Student-Centred	Effective assessment empowers students through voice and choice. It includes a variety of assessment formats such as products, observations, and conversations with the students.
Student Agency	Effective assessment guides students in understanding and articulating their learning.

EVIDENCE OF LEARNING

Assessments may be formal or informal, and they may be formative or summative. Assessment tasks vary from asking questions during a discussion to a written test at the end of the learning. Assessments of all types provide evidence for teachers to make decisions about the next steps forward in the students' learning.



The purpose of formative assessment is to gather evidence of what the students know, can do and understand during the learning process. Teachers are encouraged to obtain information from a variety of sources, such as products, observations and conversations with the students. Many students can demonstrate their understanding more accurately using some formats over others. For example, some students do better with a written format, and others may prefer to draw pictures or use a model to explain their thinking. When students can show their mathematical understanding in multiple ways, teachers can accommodate different learning styles and strengths.

Effective Assessment Tools - Criteria

When choosing or creating assessments, it is important that the assessments elicit the right kind of evidence. Assessment tools, whether formative or summative, that have been created by publishers or others, don't always address the targeted learning goals or even the correct mathematics curriculum. When this is the case, it is recommended that teachers create assessments for their own students, such as discussions, questioning and performance tasks. The following is a list of further criteria that can help determine the quality of an assessment task.

Effective assessment tasks:

- Directly relate to the learning intentions.
- Are explicit about what learners are required to do.
- Include clear assessment criteria.
- Are fair to all students including those with additional needs.
- Are scored based on rubrics.
- Are appropriate to where learners are in their learning.

FORMATIVE & EMBEDDED

Examples

Diagnostics	**Pretests** • Used to inform instruction. • Does not need to be lengthy. • Use resources on hand. • Make notes about individual strengths and class trends. • Limit the number of concepts assessed. **Screeners** • Leaps and Bounds • Class Discussions • KWL Charts • Concept Maps • Show What You Know
Conferences & Conversations	• Can expose student's misconceptions and provide a sesnse for how students are thinking. • Informs instruction and direction for future lessons. • Can gather quick, individual responses. • Student-led conferences showcase growth and can allow for student voice and choice.
Interviews	• Use what we know about students' cognition to design an assessment or future lessons. • Probe for understanding, identifiy misconceptions. • Provides accurate information about student understandings.
Products	• Written products • Computations and short-answer questions • Multi-step and open-ended questions • Journals, learning logs, exit cards • Quizzes, tests • Questions
Routines	Many of the recommended number routines can be used as assessment tools when combined with another strategy such as observation or exit task.
Observations	• Observing and listening to students, providing feedback. • Anecdotal notes • Checklists • Questioning • Reveals the logic behind the thinking, allows students to explain thinking
Open Questions	• Allow students to demonstrate their critical thinking. • Allow students to approach from a variety of backgroudn knowledge and strategies.
Peer & Self Reflection	• Journal Entries • Verbal Check-ins • Quick responses
Parallel Tasks	Offers same concept at different entry levels and at differing complexities. Students choose the task they wish to complete.
Rubrics	Scale based on predetermined criteria with two important fuctions. 1- Permits the student to see what is central to excellent performance. 2- Provides teacher with scoring guidelines that support analysis of student work. • Generic Rubrics are used for more than one task or subject area. • Task-Specific Rubrics are created specifically for one subject, topic, or task.
Tasks	• Performance-based • Individual, group, or partner Translation Task • Demonstrate understanding of a single problem or concept in an open way.
Portfolios	• Record growth over time and allow students to go back and reflect on their own learning progress. • Video, photos • Student narratives • Set of problems • Piece of writing or drawing

Additional Resources

General

- **About Teaching Mathematics**
  Marilyn Burns
- **Becoming the Math Teacher You Wish You'd Had**
  Tracy Johnston Zager
- **Big Ideas from Dr. Small**
  Marian Small
- **Children's Mathematics: Cognitively Guided Instruction**
  Thomas P Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi, & Susan B. Empson
- **Elementary and Middle School Mathematics: Teaching Developmentally**
  John Van de Walle, Karen Karp, & Jennifer Bay-Williams
- **Making Math Meaningful to Canadian Students, K-8**
  Marian Small
- **Mathmatical Mindsets**
  Jo Boaler
- **Solving for Why: Understanding, Assessing, and Teaching Students Who Struggle with Math, Grades K-8**
  John Tapper
- **Teaching Student-Centered Mathematics series**
  Pearson
- **Visible Learning For Mathematics, Grades K-12: What Works Best to Optimize Student Learning**
  John Hattie, Douglas Fisher, Nancy Frey, Linda Gojak, Sara Moore, & William Mellman

Assessment

- **The Formative 5**
  Francis (Skip) Fennell, Beth McCord Kobett, & Jonathan A. Wray
- **Informative Assessment, Grades K-6: Formative Assessment to Improve Math Achieve**
  Jeane M. Joyner & Mari Muri
- **Math That Matters: Targeted Assessment and Feedback for Grades 3–8**
  Marian Small
- **Math Running Records in Action**
  Dr. Nicki Newton
- **Mathematics Formative Assessment: 50 More Practical Strategies for Linking Assessment, Instruction, and Learning, Volume 2**
  Page Keeley & Cheryl Tobey
- **Prime: Number and Operations**
  Nelson

Classroom Structures/Frameworks

- **Guided Math: A Framework for Mathematics Instruction**
  Laney Sammons
- **Guided Math in Action**
  Dr. Nicki Newton
- **Math Workshop**
  Jennifer Lempp
- **Math Work Stations**
  Debbie Diller
  Math Workshop in Action
  Dr. Nicki Newton
- **Math Workstations in Action**
  Dr. Nicki Newton

Instructional Routines/Number Talks

- **The 5 Practices in Practice: Successfully Orchestrating Mathematical Discussion in your Middle School Classroom**
  Margaret (Peg) Smith & Miriam Gamoran Sherin
- **5 Practices for Orchestrating Productive Mathematics Discussions**
  Margaret Schwan Smith
- **Digging Deeper: Making Number Talks Matter Even More**
  Ruth Parker & Cathy Humphreys
- **High-Yield Routines for Grades K-8**
  Ann McCoy, Joann Barnett, & Emily Combs
- **Intentional Talk: How to Structure and Lead Productive Mathematical Discussions**
  Elham Kazemi, Allison Hintz, & Megan Franke
- **Making Number Talks Matter**
  Cathy Humphreys & Ruth Parker
- **Number Sense Routines: Building Numerical Literacy Every Day in Grades K–3**
  Jessica F Shumway
- **Number Sense Routines: Building Numerical Literacy Every Day in Grades 3-5**
  Jessica F Shumway
- **Number Talks: Fractions, Decimals, and Percentages**
  Sherry Parrish & Ann Dominick
- **Number Talks: Whole Number Computation, Grades K-5**
  Sherry Parrish

Additional Resources

Mathematical Concepts

- **Beyond Invert and Multiply, Grades 3-6: Making Sense of Fraction Computation**
  Julie McNamara
- **Beyond Pizzas & Pies, Grades 3-5, Second Edition: 10 Essential Strategies for Supporting Fraction Sense**
  Julie McNamara & Meghan M Shaughnessy
- **Building Proportional Reasoning Across Grades and Math Strands, K-8**
  Marion Small
- **Extending Children's Mathematics: Fractions & Decimals: Innovations In Cognitively Guided Instruction**
  Susan B. Empson & Linda Levi
- **How Children Learn Number Concepts**
  Kathy Richardson
- **Learning and Teaching Early Math: The Learning Trajectories Approach**
  Douglas H. Clements & Julie Sarama
- **Math Expressions: Developing Student Thinking and Problem Solving Through Communication**
  Cathy Marks Krpan
- **Multiplicative Thinking for Grades 3-8**
  Carole Fullerton
- **Proportional Reasoning in Grades 4-8**
  Carole Fullerton
- **Taking Shape: Activities to Develop Geometric and Spatial Thinking**
  Joan Moss, Catherine D. Bruce, Bev Caswell, Tara Flynn, & Zachary Hawes
- **Uncomplicating Fractions to Meet Common Core Standards in Math, K-7**
  Marian Small
- **Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction**
  Catherine Twomey Fosnot & Maarten Dolk

Open Questions/Engaging Tasks

- **Good Questions: Great Ways to Differentiate Mathematics Instructions**
  Marian Small
- **Mindset Mathematics: Visualizing and Investigating Big Ideas, Grade 3**
  Jo Boaler, Jen Munson, & Cathy Williams
- **Mindset Mathematics: Visualizing and Investigating Big Ideas, Grade 6**
  Jo Boaler, Jen Munson, & Cathy Williams
- **More Good Questions: Great Ways to Differentiate Secondary Mathematics Instructions**
  Marian Small & Amy Lin
- **Open Questions for Rich Math Lessons**
  Marian Small

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